

Session 3: Time Value of Money

Foundations of Finance

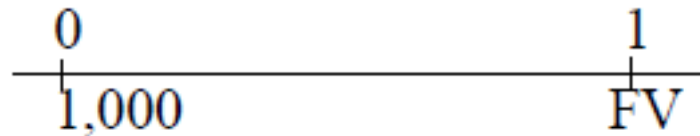
Fall 2025

Outline

- Future value (FV), present value (PV), and yield (r)
- Examples
 - Single payment securities, e.g., zero coupon bonds
 - Multiple payment securities, e.g., annuities and perpetuities

Future Value: One Period

- What is the value of \$1000 invested for one year at an annual rate of 5%?

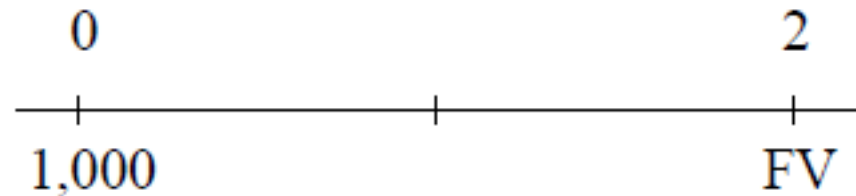


$$FV = PV(1+r) = 1000(1+5\%) = 1000(1.05) = 1050$$

- Principal plus (simple) interest

Future Value: Multiple Periods

- What is the value of \$1000 invested for two years at an annual rate of 5%?



$$FV = PV(1+r)(1+r) = 1000(1.05)^2 = 1102.50$$

$$FV = 1000 + 2(5\%)1000 + 5\%(5\%)1000$$

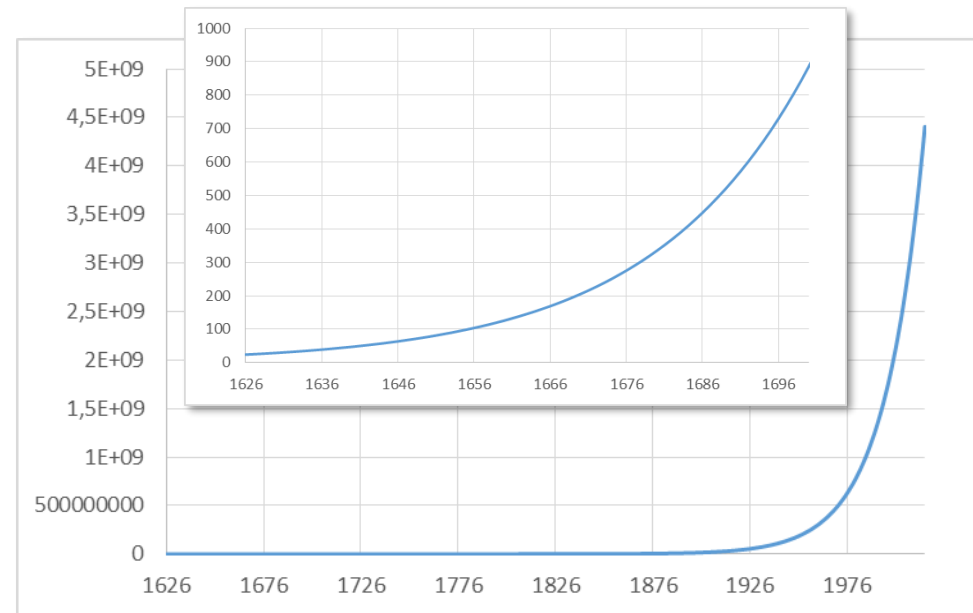
- Principal plus simple interest plus compound interest
- More generally: $FV = PV(1+r)^t$

Markets: New York Real Estate

- The Dutch West India Company dispatched the first permanent settlers to Manhattan Island in 1624. In 1626, the fledgling town's governor, Peter Minuit, bought Manhattan from the Canarsie tribe for \$24 worth of beads and trinkets. Locals sometimes cite this transaction as one of the last real estate bargains in New York.
- How big of a bargain was it at a 5% interest rate?

Assume 5% interest rate, the future value of \$24 is $24(1.05)^{397} = \$6.2 \text{ billion}$ (6,199,590,005)
A lot of money, but a good deal for the Dutch.

How much of this is simple interest?
 $24(0.05)(397) = \$476.4$ (The power of compounding!!)



Present Value: One Period

- To receive \$1000 in one year, how much should I invest today at a rate of 5%?

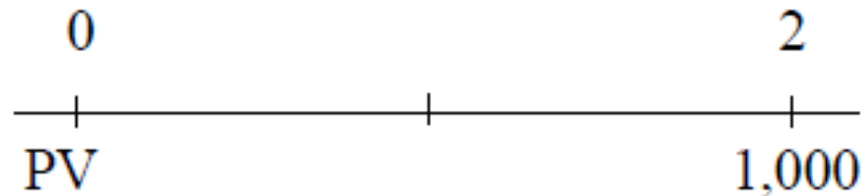


$$PV = FV/(1+r) = 1000/(1+5\%) = 1000/1.05 = 952.38$$

- This formula should look very familiar!

Present Value: Multiple Periods

- To receive \$1000 in two years, how much should I invest today at a rate of 5%?

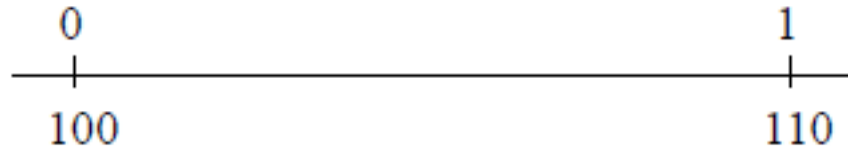


$$PV = [FV/(1+r)]/(1+r) = 1000/1.05^2 = 907.03$$

- In general: $PV = FV/(1+r)^t$

Discount Rate/Yield

- One period
 - If you invest \$100 and receive \$110 in 1 year, what rate are you earning?



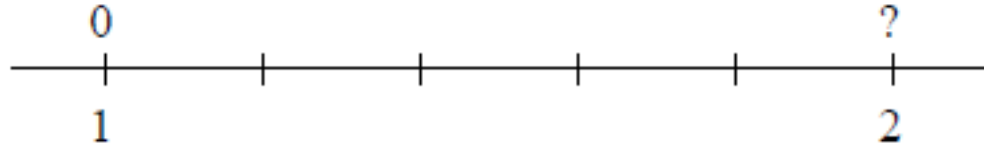
$$r = (FV/PV) - 1 = (110/100) - 1 = 0.1 = 10\%$$

- Multiple periods
 - If an investment offers to double your money in 5 years, what rate are you earning?

$$r = (FV/PV)^{1/t} - 1 = (2/1)^{1/5} - 1 = 14.87\%$$

Time

- At a rate of 5% how long will it take to double your money?



$$t = \log(FV/PV)/\log(1+r)$$

$$t = \log(2/1)/\log(1.05) = 14.21 \text{ years}$$

Recap

- Given 3 you can always find the 4th

$$FV = PV(1+r)^t$$

$$PV = FV/(1+r)^t$$

$$r = (FV/PV)^{1/t} - 1$$

$$t = \log(FV/PV)/\log(1+r)$$

Zero Coupon Bonds

- What is a zero coupon bond?

A bond that pays no interest/coupon, only the face value at maturity.

- Where do zero coupon bonds come from?
 - Issued in primary markets (Treasury bills)
 - Stripping of coupon bonds (example of “financial engineering”)

Zero Coupon Bond

$F=100$, $r=5\%$, $t=10$

Price?

Pricing Zero Coupon Bonds

$$\text{Price} = \text{PV} = F/(1+r)^t$$

$$F=100, r=5\%, t=10 \rightarrow \text{PV}=100/(1.05)^{10}=61.39$$

- $t=5 \rightarrow \text{PV} =$

$$t=1 \rightarrow \text{PV} =$$

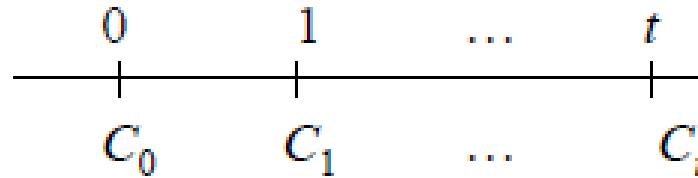
Pull to par

- $r = 4\% \rightarrow \text{PV} =$

Lower interest rates \rightarrow higher bond prices

Multiple Payments

- Present value of a series of cash flows C_0, C_1, \dots, C_t



- Future value of the same cash flow stream

$$PV = C_0 + C_1 \frac{1}{(1+r)} + \dots + C_t \frac{1}{(1+r)^t}$$

$$FV_t = C_0(1+r)^t + C_1(1+r)^{t-1} + \dots + C_t$$

Multiple Payments: An Example

- What is the PV of a \$1,000 par, 2-year, 5% coupon bond (annual payments) at a 6% interest rate?

0	1	2
PV	50	1,000+50

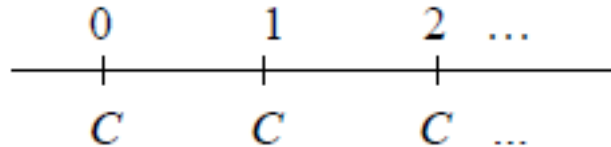
$$PV = 50/1.06 + 1050/1.06^2 = 47.17 + 934.50 = 981.67$$

This is a discount bond (price < par)! Why?

- What is the present value at 5%?

Perpetuities

- Definition: Pays a fixed cash flow, C , every period, forever (starting at time 1)



- Pricing: $PV = C/r$

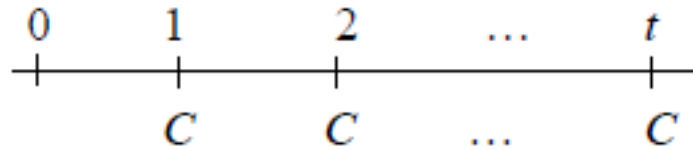
$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} \dots + \frac{C}{(1+r)^t} + \dots$$

Perpetuity: Example

- Suppose that maintenance of your grave costs \$100 every year, forever
- The interest rate is 5% per year
- How much money should you leave your trustee?

Annuities

Definition: Pays a fixed cash flow, C , for t periods (starting at time 1)



$$\begin{aligned} PV &= \frac{C}{(1+r)} + \frac{C}{(1+r)^2} \dots + \frac{C}{(1+r)^t} \\ &= C \frac{\left[1 - \frac{1}{(1+r)^t} \right]}{r} = C \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right] \end{aligned}$$

Annuity: Example

What car can you afford?

- You have no large amount of cash
- You can afford \$632 per month
- You can borrow at an interest rate of 1% per month
- You want to have paid the loan in full in 48 months

$$\begin{aligned} PV &= C \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right] \\ &= 632 \left[\frac{1}{0.01} - \frac{1}{0.01(1+0.01)^{48}} \right] = 24,000 \end{aligned}$$

Interest Rates and Prices

- For bonds?

Inverse relationship between price and interest rate. You give me the interest rate and I give you the price or vice versa

- For perpetuities?

Inversely proportional. Double interest \Rightarrow half the price. This is unique to perpetuities

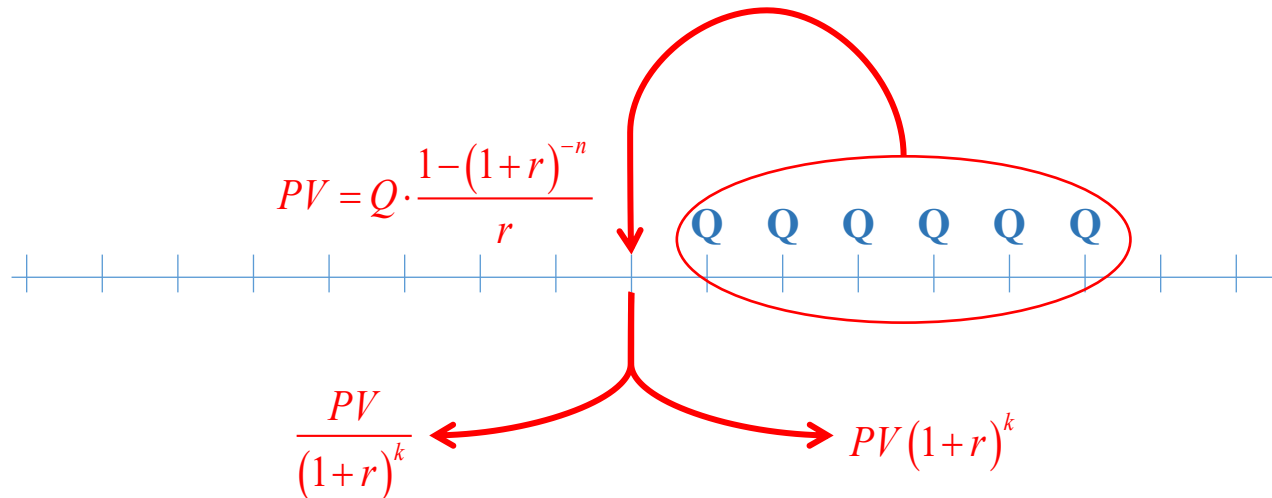
- For stocks?

inverse relation between price and discount rate (holding expected cash flows fixed!)

Conclusion

Time value of money is easy:

There is only one formula!
(perhaps 2)



Assignments

- Reading
 - RWJ: Chapter 5.3, 8.1, 8.4
 - Handout 01 - Numerical Example - Valuating a zero coupon bond
 - Handout 02 - Annuities and Perpetuities
 - Handout 03 - Continuous Compounding
- Problems: 5.12-5.15, 5.43, 8.5-8.7, 8.25
- Assignments
 - Problem Set 1 due 15th September