Session 10: The CAPM I

Fall 2025

Outline

- Assumptions
- The tangency portfolio
- The required returns on individual assets

Monday		Wednesday		Friday	
		Session 10, October 1st	The CAPM. Equilibrium asset pricing		
Session 11, October 6th	The CAPM II Applying the CAPM	Session 12, October 8th	Market Efficiency Problem set 3		
		Session 13, October 15th	Review/Problem Session	Session 14, October 17th	Midterm Exam

Midterm: October the 17th

Introduction: CAPM

- Equilibrium model that
  - Explains the relationship between risk and expected return
  - Gives optimal portfolio choices
  - Underlies much of modern finance theory and real-world financial decision making
- Derived using Markowitz's principles of portfolio theory (with additional simplifying assumptions)
- Sharpe, Lintner and Mossin are researchers credited with its development

Key Questions

1. Which portfolios should investors hold in equilibrium?

2. What is the equilibrium required return, E[R], or the equilibrium price of an individual stock (asset)?

CAPM Assumptions

1. The market is in a competitive equilibrium

   Demand=Supply → equilibrium price. i.e.,

       if demand>supply, ↑price.

       if demand<supply, ↓price.

   Investors take prices as given and no one player is big enough to manipulate the market (no monopolists).

2. Common, single-period investment horizon

   Static Problem

3. All assets are tradable

   Every investor can invest in the same assets

   The portfolio of all the traded assets the market portfolio

CAPM Assumptions

4. No transaction costs, no taxes

   Investors pay neither taxes on returns nor transaction costs (commissions and service charges) on trades.

   Bid-ask spread assumed to be zero.

   Investors have access to unlimited risk-free borrowing and lending. The interest rate to borrow and lend is the same.

5. Investors are rational, mean-variance optimizers

   Investors only care about mean returns and standard deviations

   They all choose optimal portfolios to get on the highest possible indifference curve given the EF.

6. Homogeneous expectations

   All investors have the same estimates/views of means, variances, and correlation across assets. As a result, all investors face the same efficient frontier.

CAPM Assumptions

- Assumptions are required for simplification $\rightarrow$ Powerful insights.
- Basically amount to the fact that investors are as similar to each other as possible. We only allow them to differ in terms of their wealth and their risk aversion.
- Some assumptions can be relaxed, and CAPM still holds.
- If many assumptions are relaxed, generalized versions of the CAPM apply.

The Tangency Portfolio?

- Recall from portfolio theory
  - All investors should have a (positive or negative) fraction of their wealth invested in the risk-free security
  - The rest of their wealth is invested in the tangency portfolio
  - The tangency portfolio is the same for all investors (homogeneous expectations)

- In equilibrium: supply = demand

  → the tangency portfolio must be the portfolio of all existing risky assets, i.e., the "market portfolio"!!

The Market Portfolio

- P_i = price of one share of risky asset *i*
- n_i = number of shares outstanding for risky asset *i*
- Market Portfolio = portfolio in which each risky asset *i* has the following weight:

$$w_{iM} = \frac{P_i \times n_i}{\sum_i P_i \times n_i}$$

$$= \frac{\text{market capitalization of security } i}{\text{total market capitalization}}$$

In words, the market portfolio is the portfolio consisting of all assets (everything! value-weighted)

The Capital Market Line

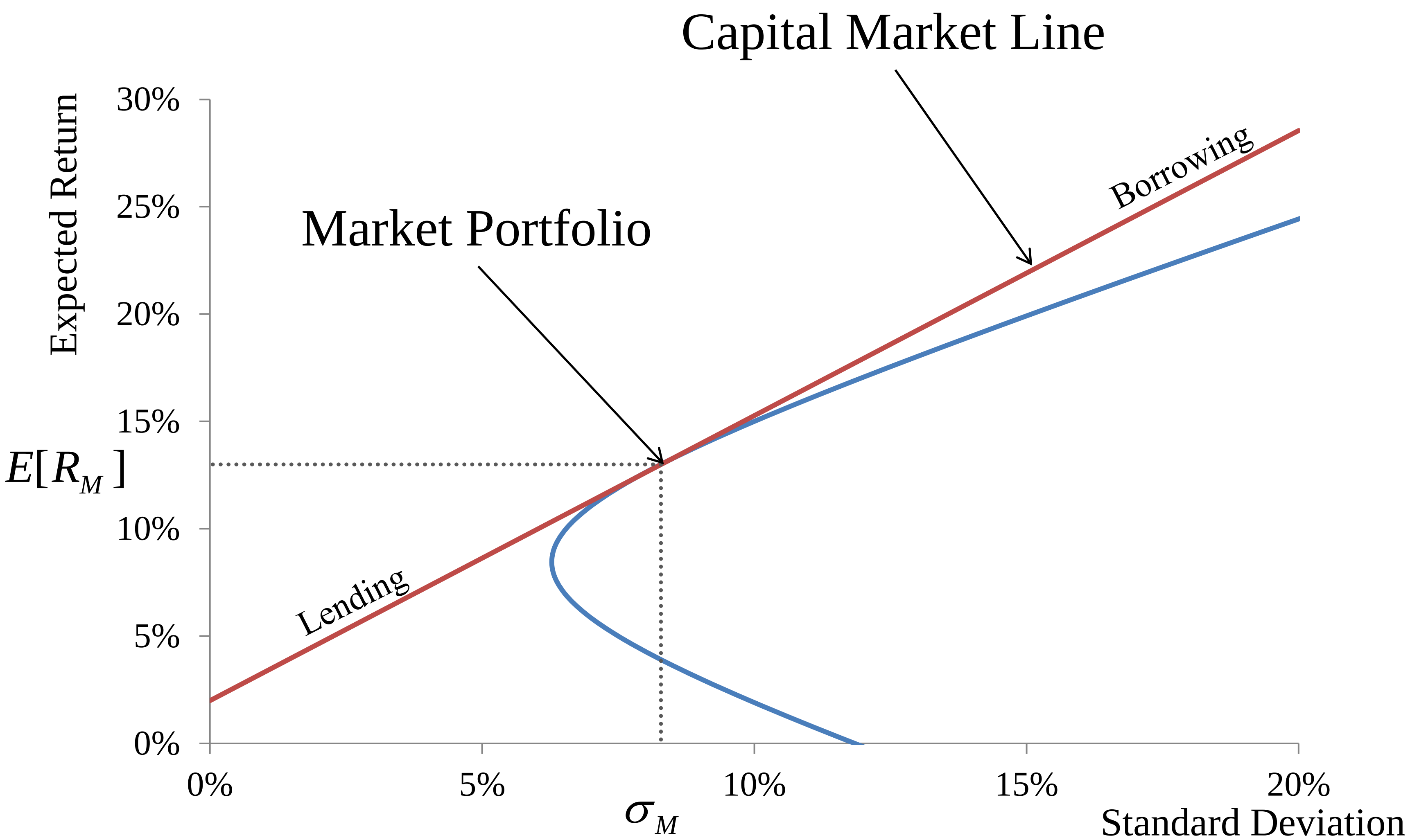
- Recall: The Capital Allocation Line (CAL) with the highest Sharpe ratio is the CAL with respect to the tangency portfolio
- In equilibrium, the tangency portfolio is the market portfolio
- The market portfolio's CAL is called the Capital Market Line (CML)
- The CML gives the risk-return combinations achieved by forming portfolios from the risk-free security and the market portfolio:

$$E[r_p] = r_f + \left(\frac{(E[r_M] - r_f)}{\sigma_M} \right) \sigma_p$$

the market **price of risk**

If everybody wakes up more risk averse tomorrow, the extra return compensation that the investors will demand per unit of risk will be higher

The Capital Market Line



CAPM demonstration

Capital Market Line (CML)

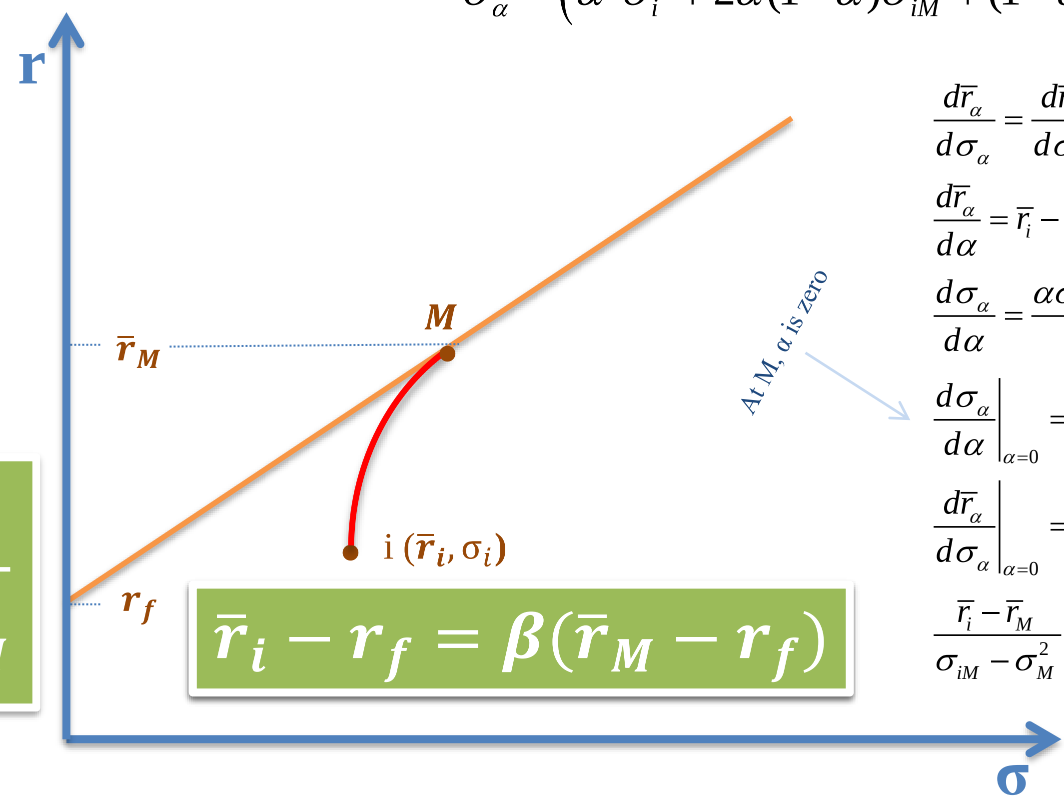
$$\bar{r} = r_f + \frac{\bar{r}_M - r_f}{\sigma_M}$$

Ok, we have a line but, what happens with individual assets?

We can make a portfolio between M (market portfolio) and i

$$\bar{r}_\alpha = \alpha \bar{r}_i + (1-\alpha)\bar{r}_M$$

$$\sigma_\alpha = \left(\alpha^2\sigma_i^2 + 2\alpha(1-\alpha)\sigma_{iM} + (1-\alpha)^2\sigma_M^2\right)^{1/2}$$



We are looking for the slope in M, that must be equal to the slope of CLM

$$\frac{d\bar{r}_\alpha}{d\sigma_\alpha} = \frac{d\bar{r}_\alpha / d\alpha}{d\sigma_\alpha / d\alpha}$$

$$\frac{d\bar{r}_\alpha}{d\alpha} = \bar{r}_i - \bar{r}_M$$

$$\frac{d\sigma_\alpha}{d\alpha} = \frac{\alpha\sigma_i + (1-2\alpha)\sigma_{iM} + (\alpha-1)\sigma_M^2}{\sigma_M}$$

At M, α is zero

$$\left.\frac{d\sigma_\alpha}{d\alpha}\right|_{\alpha=0} = \frac{\sigma_{iM} - \sigma_M^2}{\sigma_M}$$

$$\left.\frac{d\bar{r}_\alpha}{d\sigma_\alpha}\right|_{\alpha=0} = \frac{\bar{r}_i - \bar{r}_M}{\sigma_{iM} - \sigma_M^2}\sigma_M$$

Both slopes are equal

$$\frac{\bar{r}_i - \bar{r}_M}{\sigma_{iM} - \sigma_M^2}\sigma_M = \frac{\bar{r}_M - \bar{r}_f}{\sigma_M}$$

$$\beta = \frac{\sigma_{iM}}{\sigma_M^2}$$

$$\bar{r}_i - r_f = \beta(\bar{r}_M - r_f)$$

The Required Return

- The CAPM is most famous for its prediction concerning the relationship between risk and expected return for individual securities:

$$E[r_i] = r_f + \frac{\partial \sigma_M}{\partial w_i} \left[\frac{E[r_M] - r_f}{\sigma_M} \right]$$

- The extra return is proportional to the risk contribution of that security to the overall market
- This relationship must hold for every security

The Risk Contribution

What is $\dfrac{\partial \sigma_M}{\partial w_i}$?

The risk contribution depends on the covariance with all other assets (see handout)

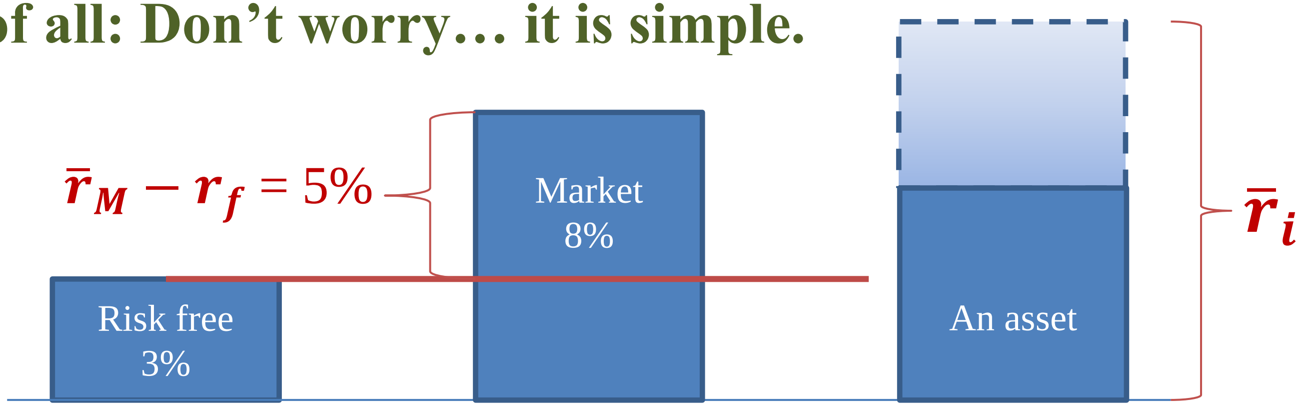
$$R_i = \alpha_i + \beta_i R_M + e_i$$

$$\beta_i = \frac{\text{cov}(R_i, R_M)}{\sigma_M^2} = \frac{\text{cov}(r_i, r_M)}{\sigma_M^2}$$

$$\frac{\partial \sigma_M}{\partial w_i} = \beta_i \sigma_M$$

Understanding CAPM

First of all: Don't worry… it is simple.



$$\bar{r}_i = r_f + \beta(\bar{r}_M - r_f) = 3\% + 5\%.\ \beta$$

$$\beta = \frac{\sigma_{iM}}{\sigma_M^2}$$

Generally speaking: we expect aggressive companies or highly leveraged companies to have high betas, whereas conservative companies whose performance is unrelated to the general market.

- $\beta < 1$: the stock price is less risky than the market (fluctuate less)
- $\beta > 1$: the stock price is more risky than the market (fluctuate more)

It looks like a correlation coefficient: $\rho_{XY} = \dfrac{\sigma_{XY}}{\sigma_X \sigma_Y}$

Security Market Line

- The model predicts that expected excess return of a security is linear its ‘beta’

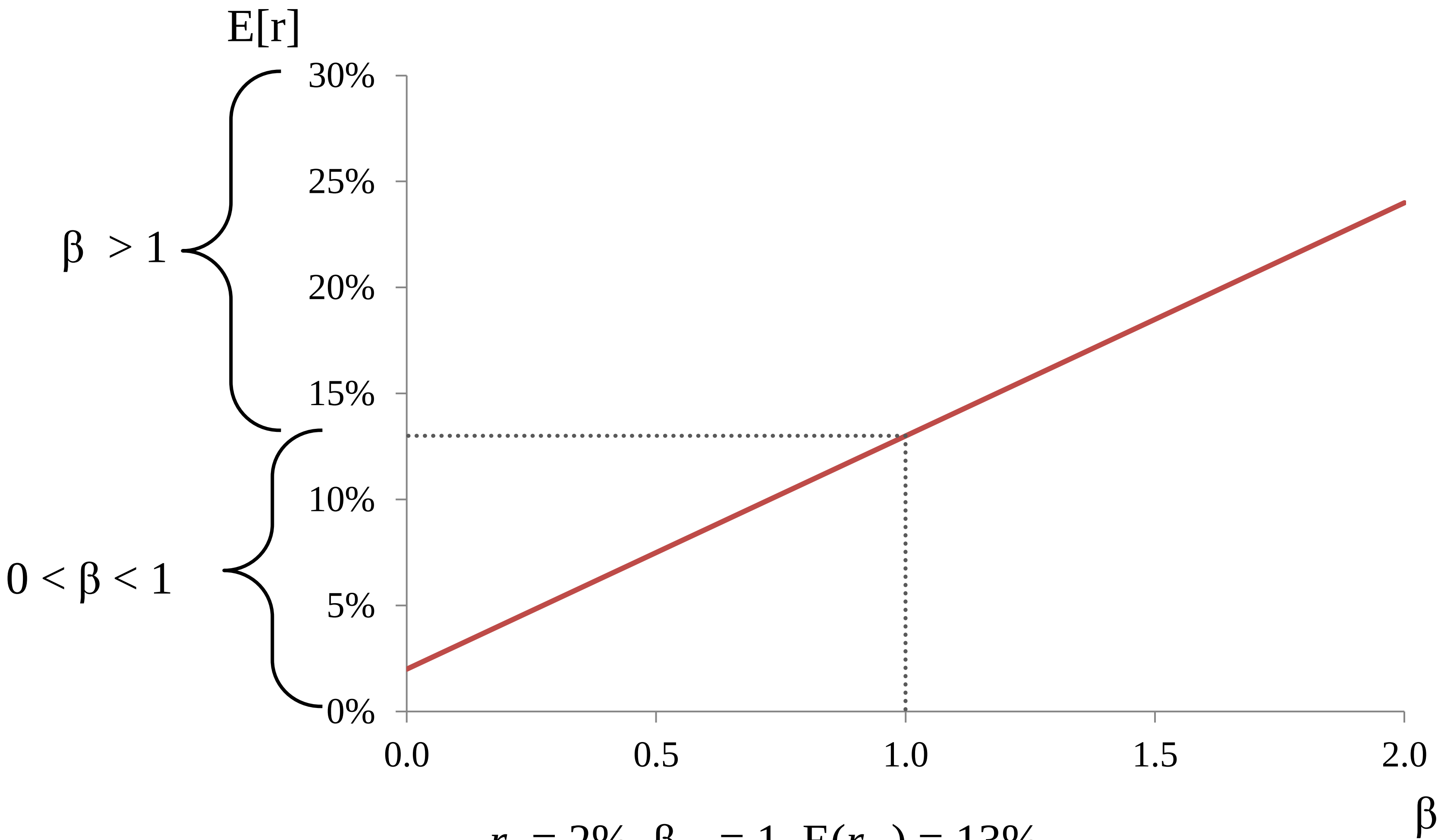
$$E[r_i] = r_f + \frac{\partial \sigma_M}{\partial w_i}\left[\frac{E[r_M] - r_f}{\sigma_M}\right]$$

$$E[r_i] = r_f + \beta_i\, E[r_M - r_f]$$

- This linear relation is the Security Market Line (SML)

- The beta measures the security’s sensitivity to market movements

Security Market Line (SML)



$r_f = 2\%$, $\beta_M = 1$, $E(r_M) = 13\%$

Risk Premium

- SML: $E[r_i] = r_f + \beta_i (E[r_M] - r_f)$
- Stock i's systematic or market risk is β_i
- Investors are compensated for holding systematic risk in the form of higher returns
- The size of the compensation depends on the *equilibrium risk premium*: $(E[r_M] - r_f)$
- The equilibrium risk premium is increasing in
  1. The variance of the market portfolio (volatile times)
  2. The degree of risk aversion of the *average* investor

Beta Intuition

The Coca-Cola Company (KO) - N

43.92 0.32(0.73%) Oct 7,

Prev Close:	43.60
Open:	43.48
Bid:	43.65 x 400
Ask:	43.90 x 400
1y Target Est:	45.54
Beta:	0.51
Next Earnings Date:	21-Oct-14

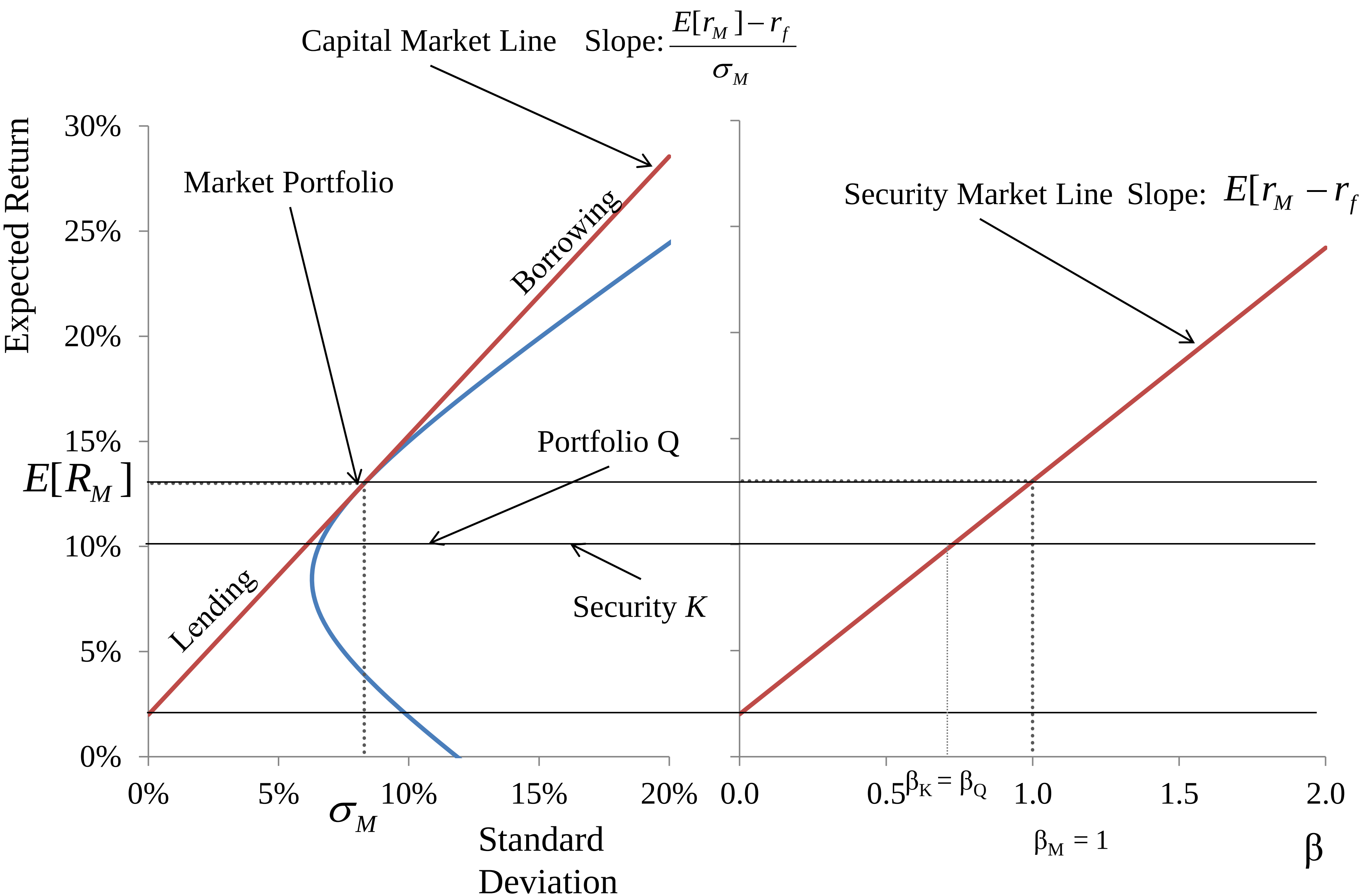
Bank of America Corporation (BAC)

16.88 0.41(2.37%) Oct 7, 4:01

Pre-Market : **16.84** 0.04 (0.24%) 6:41AM EDT

Prev Close:	17.29
Open:	17.18
Bid:	16.83 x 1000
Ask:	16.87 x 1000
1y Target Est:	18.02
Beta:	1.42
Next Earnings Date:	15-Oct-14

The CML and the SML



Conclusion

A theory of risk and return:

- How to measure risk—beta
- The price of risk—the market risk premium

Assignments

- Reading
  - BKM: Chapters 7.3-7.5
  - Problems: 7.8, CFA 7.2, 7.6-7.9
- Assignments
  - Problem Set 3 due October, 8^{th} (Lesson 12)