Session 11: The CAPM II

Fall 2025

Outline

- Systematic and idiosyncratic risk
- Implementation
- Applications of the CAPM

Systematic Risk

- The priced risk of an individual stock is the systematic risk, not the idiosyncratic risk.
- β_i measures how that stock covaries with all the other assets in the economy, or security *i*'s contribution to the total risk of the market portfolio
  → β_i measures the systematic risk of the stock
- Investors must be compensated for holding systematic risk (the CAPM)
  $E[r_i] = r_f + \beta_i (E[r_M] - r_f)$

Systematic and Idiosyncratic Risk

- The CAPM equation can be (also) written as

$$r_i = r_f + \beta_i (r_M - r_f) + e_i$$

$$\beta_i = \frac{\text{cov}[r_i, r_M]}{\sigma_M^2} \quad \text{E}[e_i] = 0 \quad \text{cov}[e_i, r_M] = 0$$

- The total risk of a security can be partitioned into two components

$$\underbrace{\sigma_i^2}_{\substack{\text{var}(r_i) \\ \text{total} \\ \text{risk}}} = \underbrace{\beta_i^2 \sigma_M^2}_{\substack{\text{systematic} \\ \text{market} \\ \text{risk}}} + \underbrace{\sigma_{ei}^2}_{\substack{\text{var}(e_i) \\ \text{idiosyncratic} \\ \text{risk}}}$$

An Example

- XYZ Internet stock has a volatility of 90% and a beta of 3. The market portfolio has an expected return of 14% and a volatility of 15%. The risk-free rate is 7%.
- What is the equilibrium expected return on XYZ stock?

- What proportion of XYZ Internet's variance is diversified away in the market portfolio?

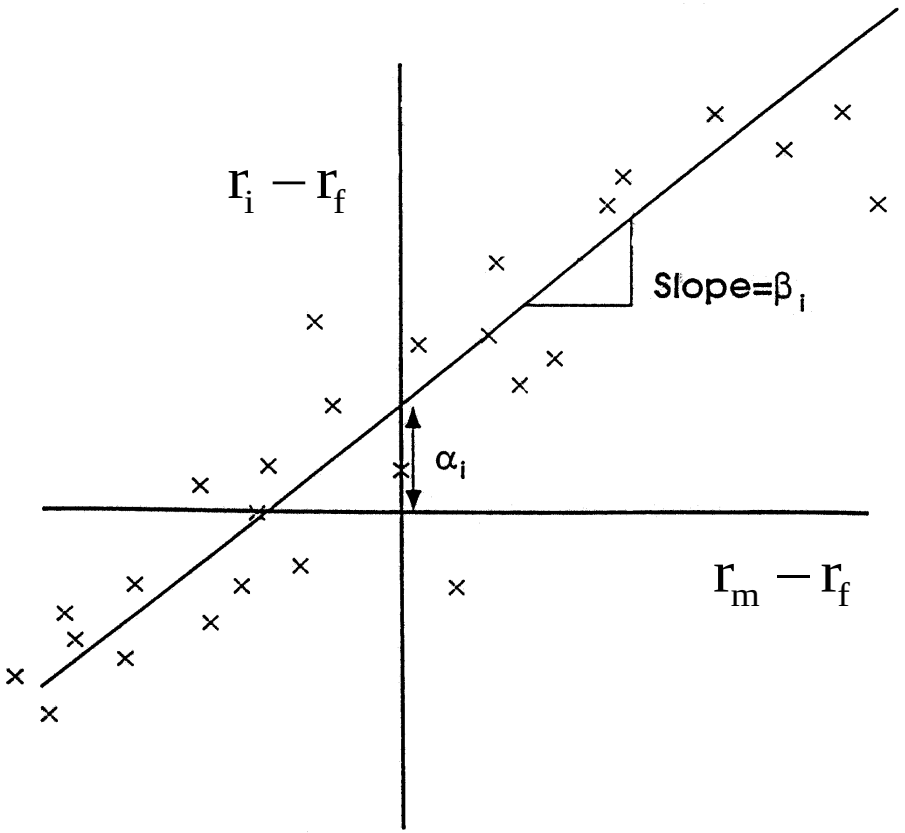
Implementation

- Beta from regression
  - ✓Industry betas
  - ✓Adjusted beta
- Risk-free rate—horizon-matched Treasury yield
- Market risk premium
  - ✓Historical average
  - ✓Forward looking
  - ✓Model generated

Security Characteristic Line (SCL)

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i (r_{M,t} - r_{f,t}) + e_{i,t}$$

- The SCL is the "regression line"
- The CAPM implies $\alpha_i = 0$



Estimating Beta

- Get data on "excess returns"

$$R_i(t-1,t) \equiv R_{i,t} = r_{i,t} - r_{f,t} \qquad R_M(t-1,t) \equiv R_{M,t} = r_{M,t} - r_{f,t}$$

Note: $r_{f,t}$ is the risk-free rate from time *t-1* to time *t*, i.e., it is the 1-period rate known at time *t-1*

- Estimate β_i by running the regression

$$R_{i,t} = \alpha_i + \beta_i R_{M,t} + e_{i,t}$$

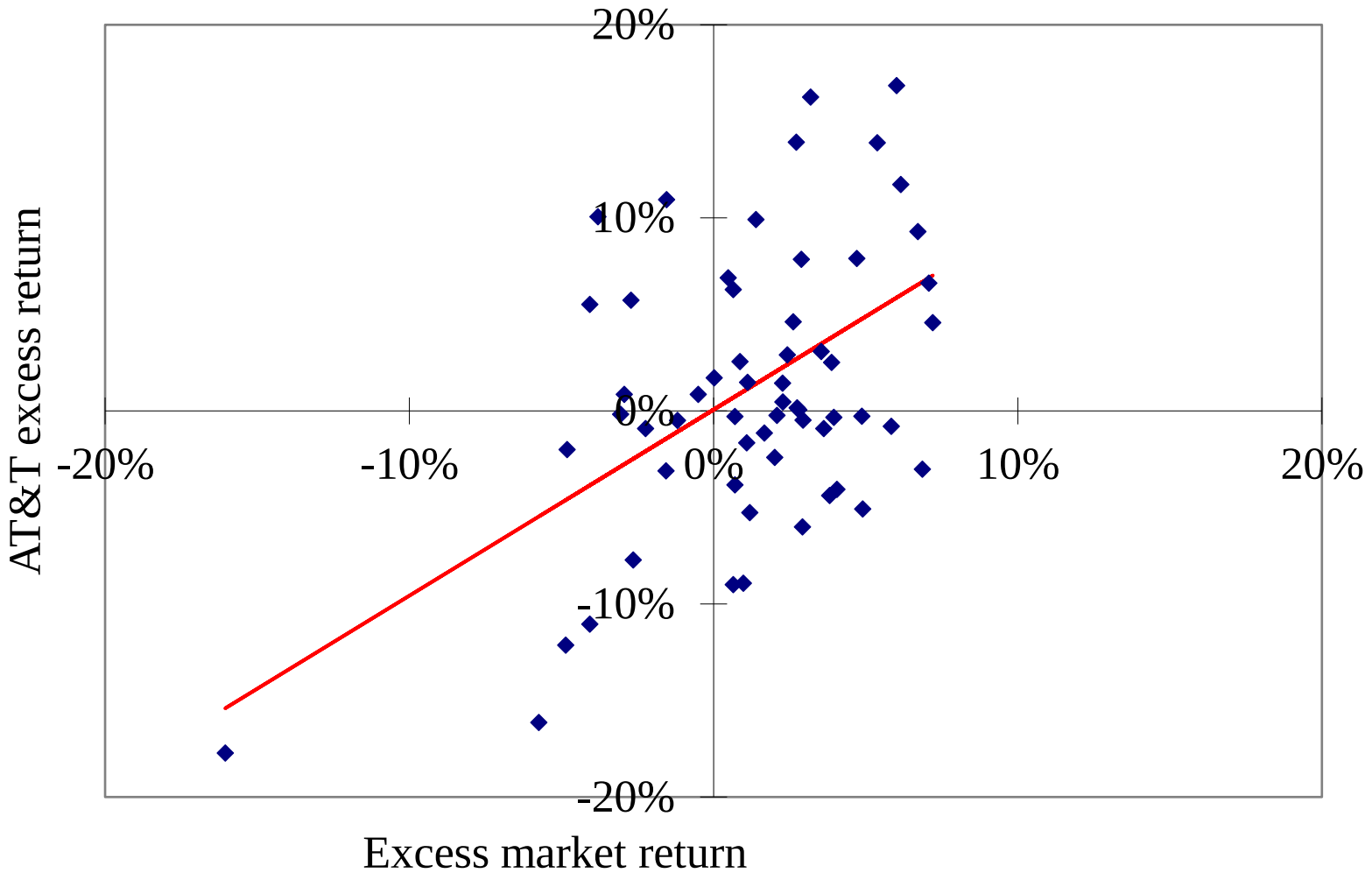
(typically, 60 months of data are used)

An Example: AT&T

- Take 5 years (1994-1998) of monthly data on AT&T returns, S&P500 returns and 1-month US T-bills
- Construct excess returns
- Run the regression, e.g., using Excel (you need to install the *Analysis ToolPak* or use =SLOPE)
- Excel output

	Coefficients	*SE*	*t Stat*	*P-value*
Intercept	0.0007	0.0091	0.0748	0.9406
X Variable 1	0.9637	0.2172	4.4366	0.0000

SCL for AT&T


20%
10%
0%
-10%
-20%
-20%
-10%
0%
10%
20%
AT&T excess return
Excess market return

Applications of the CAPM

1. Portfolio choice in equilibrium
2. “Fair” security returns
3. Benchmark for security analysis
4. Required return used in capital budgeting

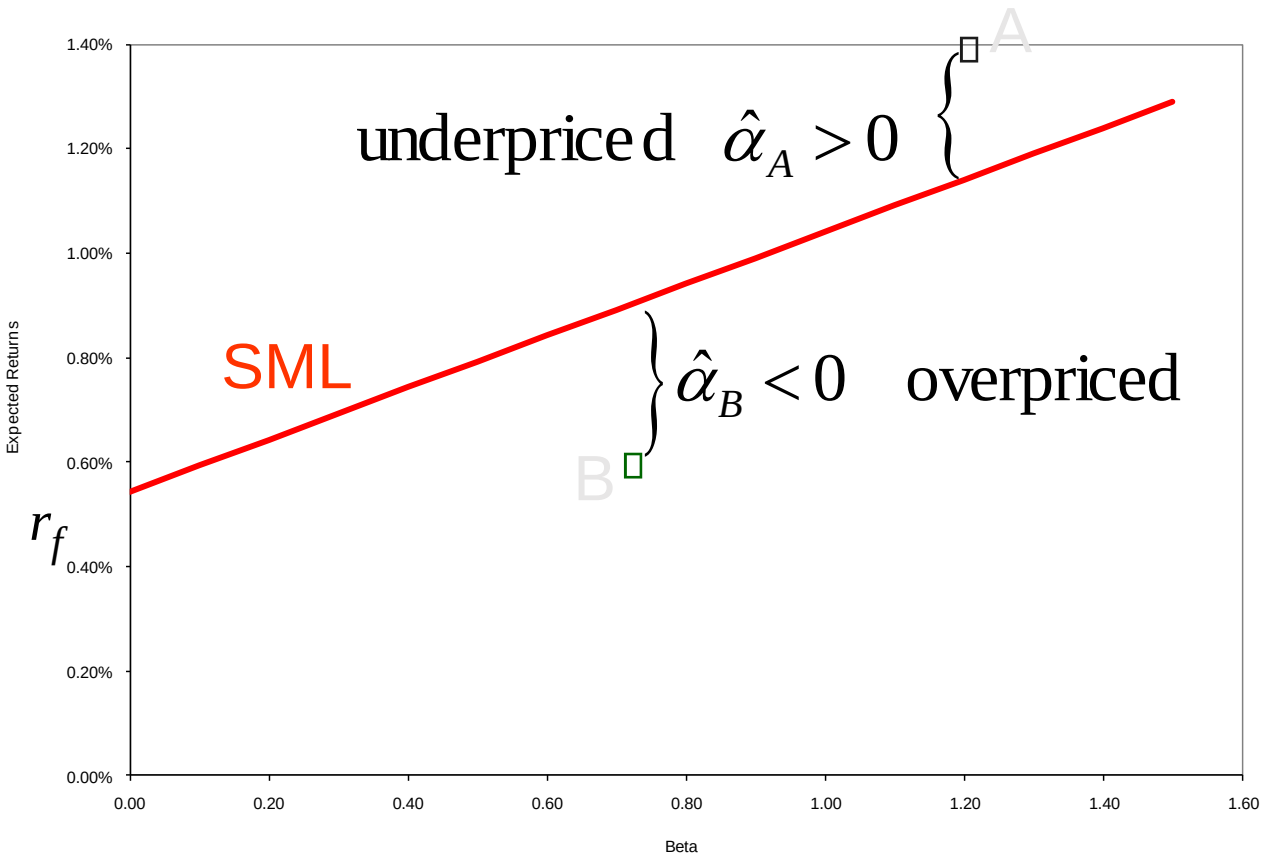
Stock Selection

- One possible benchmark for stock selection is to find assets that are cheap relative to the CAPM
- A security *i*'s alpha is defined as:

$$\alpha_i = E[r_i] - r_f - \beta_i (E[r_M] - r_f) \qquad \beta_i = \frac{\text{cov}[r_i, r_M]}{\sigma_M^2}$$

- Some fund managers try to buy positive alpha stocks and (short) sell negative alpha stocks
- The CAPM predicts that all alphas are zero

Stock Selection



Active and Passive Strategies

- An "active" strategy tries to beat the market by stock picking, market timing, etc.
- The CAPM implies that
  - ✓Security analysis is not necessary
  - ✓Every investor should just buy a mix of the risk-free security and the market portfolio, i.e., a "passive" strategy
- Grossman-Stiglitz paradox: How can the market be efficient if everyone uses a passive strategy?

Capital Budgeting and NPV

- Should a firm undertake a long-term risky project?
- Calculate Net Present Value (NPV), using CAPM to calculate required rate of return
- Manager's objective: increase value of firm
  → only undertake projects with NPV>0
- Compare with decision based on Internal Rate of Return (IRR) on project

An Example: NPV

- Business plan generates expected project cash flows in each year:

0	1	2	3	4	5
−1000	0	0	300	600	900

- Should you undertake this project?
- Discount rate? *Cost of capital* is given by CAPM

$$E[r_i] = r_f + \beta_i E[r_M - r_f]$$

- If $\beta_i = 1.75$, $r_f = 0.04$, $E[r_M]=0.12$, then $E[r_i]=0.18$
- NPV = $-1000+300/(1.18)^3 + 600/(1.18)^4 + 900/(1.18)^5$
  $= -114.54 < 0 \quad \rightarrow$ Don't undertake project!
- If $\beta_i = 0.50$, then $E[r_i]=0.08$ and NPV = $291.69 > 0$

An Example: IRR

- Where is the break-even point?
- IRR is the discount rate that sets NPV = 0

$$-1000+\frac{300}{(1+IRR)^3}+\frac{600}{(1+IRR)^4}+\frac{900}{(1+IRR)^5}=0$$

- IRR = 14.67%
- IRR rule: Undertake project if E[r_i] < IRR
- When comparing attractiveness of 2 projects, use NPV rule
- Is this analysis only appropriate for well diversified firm/project?

Summary

- The CAPM follows from equilibrium conditions in a frictionless, mean-variance economy with rational investors
- Prediction 1: Everyone should hold a mix of the market portfolio and the risk-free asset
- Prediction 2: The expected return on an individual security is a linear function of its beta, i.e., securities should be on the SML. A stock's beta can be estimated using historical data by linear regression, i.e., by estimating the SCL

$$\beta_i = \frac{\text{cov}[R_i, R_M]}{\sigma_M^2}$$

Assignments

- Reading
  - BKM: Chapters 8
  - Problems: 8.1-8.12, 8.15-8.17, 8.19-8.22, CFA 8.1-8.6
- Assignments
  - Problem Set 3 October 8th