Session 17: Arbitrage

Fall 2025

Outline

- Arbitrage definitions
- Arbitrage pricing
- Arbitrage pricing with transactions costs
- Real-world arbitrage trading

Arbitrage Definitions

- In finance *theory* an "arbitrage" is defined as: A zero-investment trading strategy that generates a sure profit, i.e., (i) no initial investment, (ii) non-negative cash flows at all times, (iii) a positive cash flow at some point
- On Wall Street "arbitrage" also often means
  - ✓ A (partially hedged) trading strategy that is expected to make a (risk-adjusted) profit
  - ✓ Statistical arbitrage, merger/risk arbitrage

Arbitrage?

"So there's an arbitrage. So what? This desk has lost a lot of money on arbitrages. Arbitrages aren't particularly great trades."

Treasury bond trader at a major Wall Street investment bank

Arbitrage Pricing

- Important insight:
  - Arbitrage opportunities cannot exist
  - If they did, arbitrageurs would trade aggressively to exploit them, thus eliminating them
  - The "*No-Arbitrage Condition*"
- Using the no-arbitrage condition we can
  - Compute restrictions on security prices
  - Price derivative securities

Implications of No Arbitrage

1. If two securities have the same payoffs, they must have the same price**:** ***Law of One Price***
2. If a portfolio has the same payoff as a security, the price of the security must be equal to the price of the portfolio (a *replicating portfolio*)
3. If a self-financing trading strategy has the same payoff as a security, the price of the security must be equal to the cost of the strategy (a *dynamic hedging/replicating strategy)*

Example: Law of One Price

- Suppose there are two kinds of risk-free, zero-coupon bonds, “CATs” and “TIGRs,” both paying a face value of $100 in 1 year
- Today’s price of
  - Chicago: CATs is $94.34 (6% YTM)
  - New York: TIGRs is $95.24 (5% YTM)
- How do arbitrageurs exploit the price difference?

Transactions Costs

- If there are transactions costs, it is more difficult to make arbitrage profits
- We cannot determine prices exactly using the No-Arbitrage Condition
- But, we can find an upper- and lower bound for the price

CATs and TIGRs Revisited

- Suppose there are two zero-coupon bonds, CATs (\$94.34) and TIGRs (\$95.24), both paying \$100 in 1 year
- The cost of shorting is \$1, and the cost of buying is \$0
- Is arbitrage still possible?
- What are the highest and lowest possible prices of TIGRs (relative to CATs) that prevent arbitrage?

- What if the cost of shorting is \$0.50 and the cost of buying is \$0.50?

Example: A Replicating Portfolio

- Suppose
  - A zero-coupon bond that matures in 1 year costs $98
  - A zero-coupon bond that matures in 2 years costs $96
  - A zero-coupon bond that matures in 3 years costs $93
- What must be the price of a 3-year, annual coupon bond with a 10% coupon rate and a face value of $100?

Arbitrage Strategy

How could you make an arbitrage profit if the coupon bond were trading at $100?

Example: A Dynamic Strategy

- Suppose
  - Today a 1-year zero-coupon bond costs $98
  - Today you can enter a contract to buy a 1-year zero-coupon bond 1 year from today for $98 (forward contract)
- What must be the price today of a 2-year zero-coupon bond?

Arbitrage Strategy

How could you make an arbitrage if the 2-year zero were trading at $95?

Real-World Arbitrage Strategies

- Index arbitrage
- Fixed-income securities, e.g., on-the-run vs. off-the-run Treasuries
- Mergers and acquisitions (“risk arbitrage”)

Conclusion

same payoffs → same price

The no-arbitrage condition (the law of one price) is a powerful tool for ***relative*** pricing.

Assignments

- Reading
  - BKM: Chapters 2.1-2.2, 10.1-10.4
  - Problems: 10.5, 10.8, 10.11, 10.13, 10.18
- Assignments
  - Problem Set 4 due <u>next</u> class