Session 18: Fixed Income I

Fall 2025

Outline

- Main features of bonds
- Yield to maturity
- Realized return

Main Features of Bonds

1. **Issuer**
   - US Treasury/Government
   - States, municipalities, and agencies
   - Corporations
   - Foreign governments (sovereign bonds)
2. **Term** (maturity)
   - Short –T-bills, CD's, commercial paper
   - Long—T-bonds, corporate bonds
3. **Price vs. par value (face value)**
   - Par bond
   - Discount bond
   - Premium bond

Features cont'd

4. **Coupon**
   - Coupon rate: total annual interest payment per dollar face value
   - Frequency (usually semi-annual)
   - Fixed or variable (floaters)
   - Nominal or inflation-indexed (TIPS)
5. **Currency**
   - Yankee bonds, Samurai bonds
   - Eurobonds
6. **Credit risk**
   - Risk-free
   - Defaultable

Features cont'd

7. **Seniority and security**
   - Senior, senior subordinated, junior, etc.
   - Secured by property, income stream, etc.
8. **Covenants**
   - Restrictions on additional issues, dividends, and other corporate actions
9. **Option provisions**
   - Callability: Issuer has the right to pay back the loan before maturity
   - Putability: Bondholder has the right to demand payment of the loan before maturity
   - Convertibility: Bondholder has the right to exchange the bond for stocks of the issuer

Yield to Maturity (YTM)

- For an annual pay coupon bond, the *YTM* is the same as the *IRR*
- Hence, *YTM* is the rate that solves

$$P_0 = \sum_{t=1}^{T} \frac{\text{coupon}_t}{(1+r)^t} + \frac{\text{face value}_T}{(1+r)^T}$$

An Example

- Suppose a 3-year bond has a face value of $1,000 and annual coupon payments of 80 (8%)
  - If the bond sells at $1000, what is the YTM?
  - If the bond sells at $900, what is the YTM?
  - If the bond sells at $1100, what is the YTM?

	Coupon Rate	Current Yield	YTM
Par bond			
Discount bond			
Premium bond			

Yield to Maturity

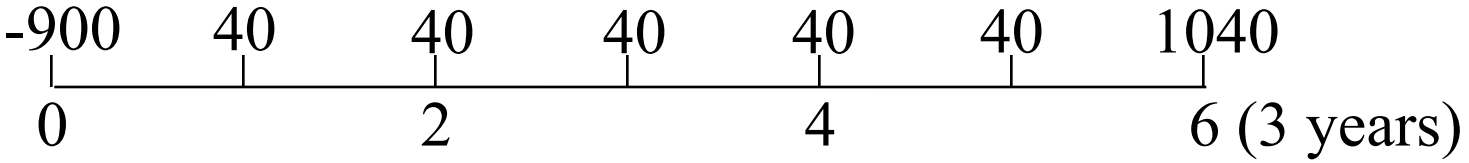
- For a semi-annual pay coupon bond, the *YTM* is computed in 2 steps:
  1. Find the semi-annual IRR, that is, the rate r that solves

$$P_0 = \sum_{n=1}^{N} \frac{\text{coupon}_n}{(1+r)^n} + \frac{\text{face value}_N}{(1+r)^N}$$

  2. The *YTM* is the corresponding annual percentage rate (APR): $YTM = 2r$
- The corresponding *effective annual yield* is $(1+YTM/2)^2 - 1$

An Example

- Suppose that a 3-year bond has a face value of 1000 and pays semi-annual coupons of 40. If the price is 900 then what is the YTM?



- Step 1: use a calculator to find the r that solves:

$$900 = \frac{40}{(1+r)^1} + \ldots + \frac{40}{(1+r)^5} + \frac{1040}{(1+r)^6} \quad \Rightarrow \quad r = 6.04\%$$

- Step 2: $YTM = 2r = 12.07\%$

$$EAY = (1+r)^2 - 1 = \left(1 + \frac{YTM}{2}\right)^2 - 1 = 12.44\%$$

Semi-Annual Pay vs. Annual Pay

- Compare the EAY of the previous examples
  - 3-year bond with face value of 1000, price of 900, and annual coupons of 80
  - 3-year bond with face value of 1000, price of 900, and semi-annual coupons of 40
- Which has a higher EAY and why?

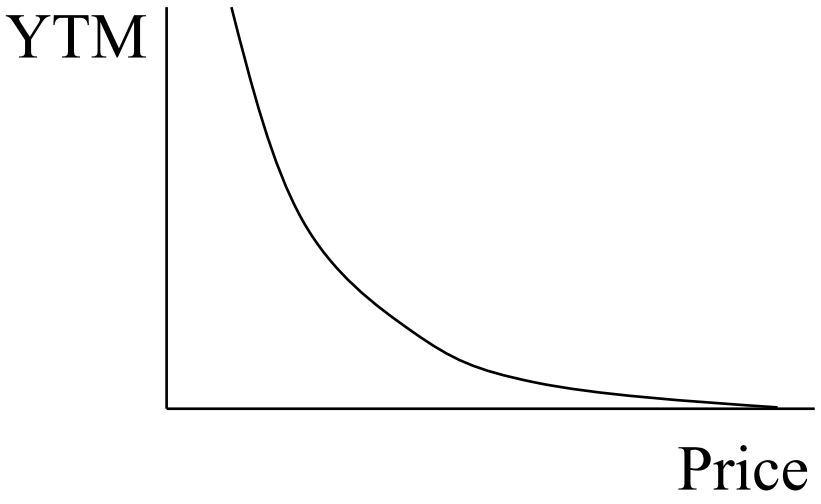
An Example

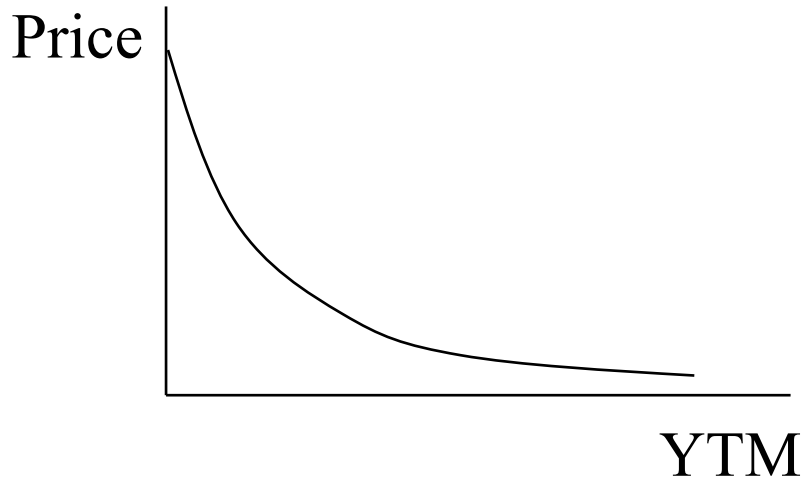
- Suppose that a 3-year bond has a face value of 1000 and pays semi-annual coupons of 40
- *Reverse question*: If the YTM is 10% then what is the price?
- Step 1: r = YTM/2 = 5%
- Step 2: substituting r = 5% into

$$P = \frac{40}{(1+r)^1} + \ldots + \frac{40}{(1+r)^5} + \frac{1040}{(1+r)^6} = 949.24$$

YTM vs. Price

Using steps 1 and 2 to go from price to YTM and vice versa we find a **negative** and **convex** relation between YTM and price





Realized Return vs. YTM

- If you buy a bond will the return (HPR) on your investment be equal to the YTM ?
- HPR = YTM if and only if
  - You can re-invest the coupons at the YTM
  - You hold the bond until maturity
- In most cases it will be different because
  - You must re-invest the coupons at a different rate
  - You sell the bond before maturity at a price that corresponds to a different yield-to-maturity. (Market yields can change.)

Realized Holding Period Return

- Suppose that
  - At time 0, you buy a bond for V_0
  - You reinvest all coupons until date t
  - At time t, you sell the bond and the reinvested coupons for a total price of V_t
- The annual holding period return (HPR) is the solution to

$$V_0(1+\text{ann. } HPR)^t = V_t$$

$$\text{ann. } HPR = \left(\frac{V_t}{V_0}\right)^{1/t} - 1$$

Example: Zero-Coupon Bond

- Suppose that a 3-year zero-coupon bond has a YTM of 5%
  - What is the bond's current price?
- Next year, the YTM changes to 7%
  - What is the price in that year?
  - What is the realized (holding period) return over one year ?
- What if the YTM in year 1 had remained 5%
  - What is the price that year?
  - What is the realized return over one year?

Example: Annual Pay Coupon Bond

- A 3-year coupon bond has face value 1000, coupons of 80, and a YTM of 8%
  - What is the bond's current price?
  - What is the future value of the bond's cash flows if you reinvest at the YTM?
  - What is the bond's ann. HPR (held to maturity)?
- If the coupon payments are reinvested at a yield of 10%
  - What is the future value?
  - What is the ann. HPR?
- What if the coupon payments are reinvested at a yield of 6%?

Yield/Return Measures for Bonds

- Coupon Rate = C/F
- Current Yield = C/P
- Yield to maturity = IRR
- Effective Annual Yield (EAY)
- Holding Period Return (HPR)

Assignments

- Reading
  - BKM: Chapter 10.6
    - Handout: Note on Forward Rates
    - Handout: Equilibrium Term Structure Under the
    - Expectations Theory
  - Problems: 10.34-10.35, 10.37, 10.39-10.41
- Assignments –Problem Set 5 – 14th November