Session 19: Fixed Income II

Fall 2025

Outline

- Forward rates
- The yield curve
- Theories
  - The expectations hypothesis
  - The liquidity preference theory

Forward Rates

- A firm foresees the need for short-term funds one year from now but is worried about the interest rate rising. Can they "lock in" a rate for a one-year loan, *starting one year from now*?
- A company will receive a payment next year and must make a payment two years from now. The company is worried about the reinvestment risk related to the incoming payment. Can the company lock in a lending rate, *starting one year from now?*
- A *forward rate* is an interest rate on a future loan that is fixed today.
- The forward rate for 1-year lending in year t (starting at time $t-1$, ending at time t) is denoted f_t

Engineering Forward Rates

- Suppose that
  - A 1-year zero has a YTM of 2%
  - A 2-year zero has a YTM of 3%
- What are the prices of these bonds?

- What is the implicit interest rate in the second year? (This interest rate is the forward rate.)

Engineering Forward Rates cont'd

How can you trade these bonds to replicate a loan between year 1 and year 2 ? ("synthetic" loan.)

Forward Rates

- The forward rate is determined by no-arbitrage

$$f_n = \frac{P_{n-1}}{P_n} - 1 = \frac{(1+y_n)^n}{(1+y_{n-1})^{n-1}} - 1$$

- Example with n = 2 (and rewriting)

$$(1+y_2)^2 = (1+y_1)(1+f_2)$$

- Forward rate = interest rate that would need to prevail in second year to make the return on long- and short-term investments equal

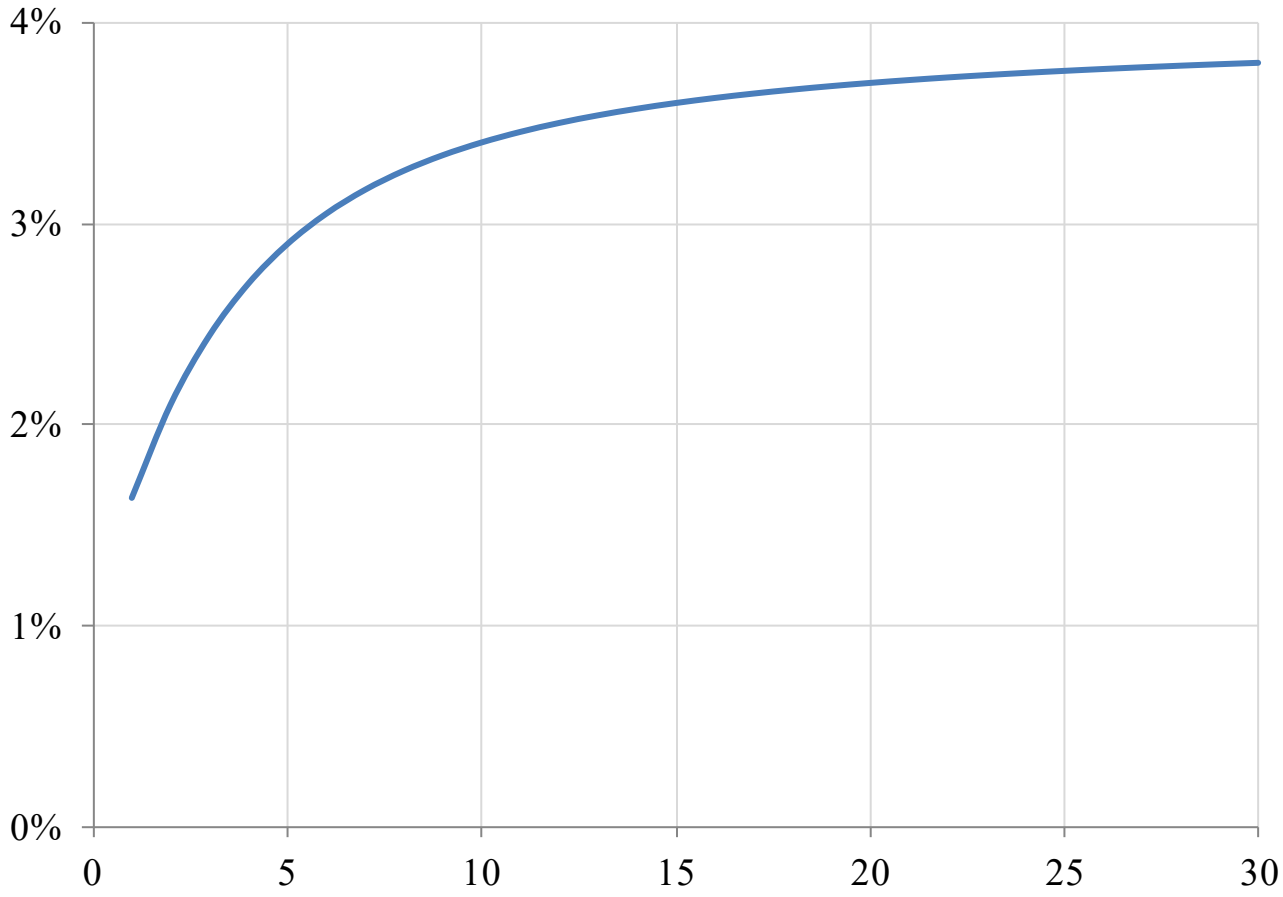
Forward Contracts and Futures

- Forward rates are also traded directly
  - FRAs: forward rate agreements
  - Eurocurrency interest rate futures
  - Bond futures
- We will come back to this in our class on futures

The Yield Curve

- The collection of YTM of zero-coupon bonds has many names
  - The term structure of zero-coupon bond yields
  - The term structure of (spot) interest rates
  - The yield curve

Yield Curve



Yield Curve

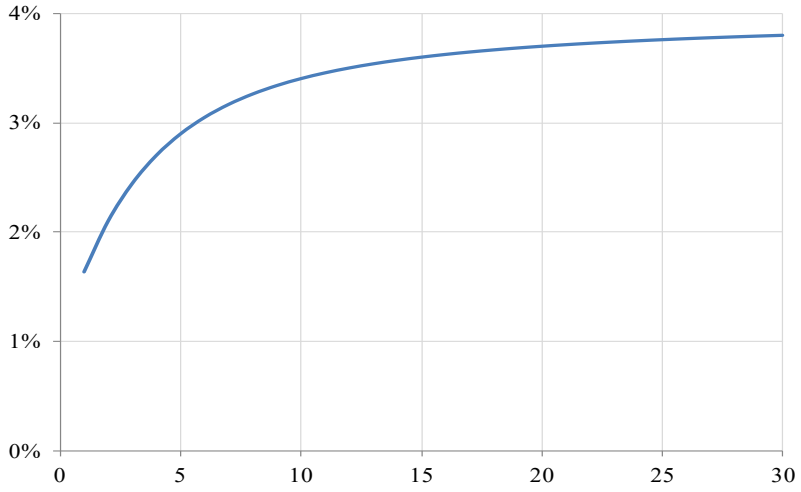
Dynamic Yield Curve – Compared to SP500

http://stockcharts.com/freecharts/yieldcurve.php

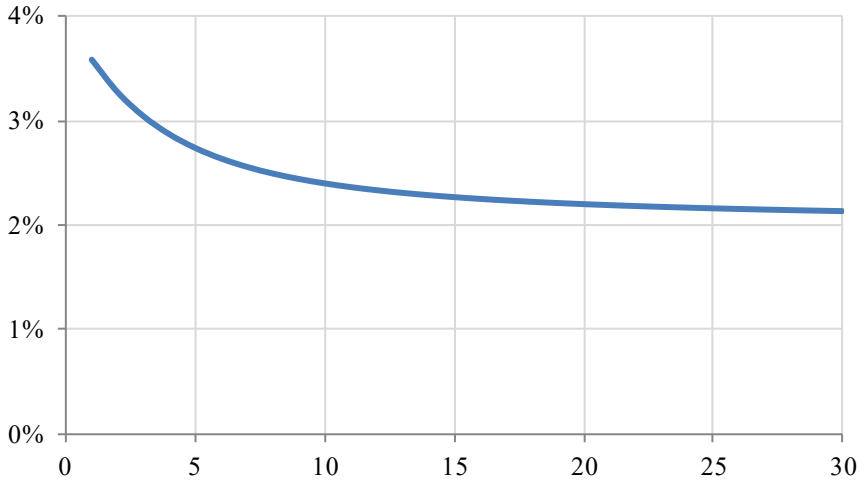
Yield Curve

Typical shapes of the yield curve:

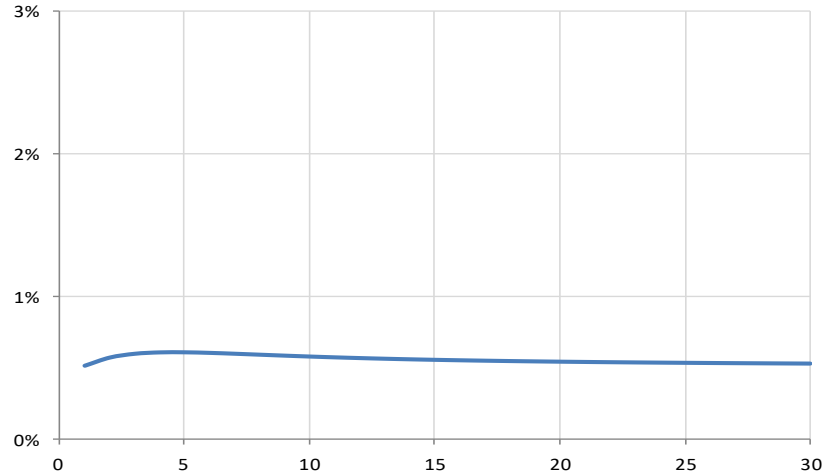
Upward sloping (most typical)



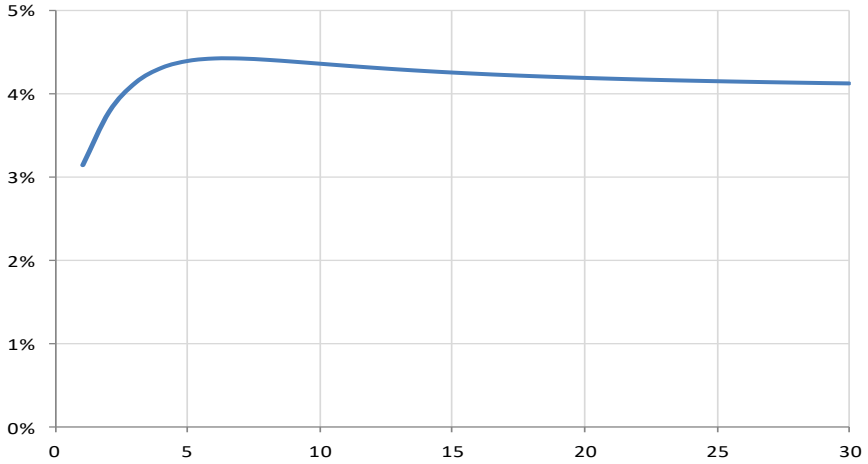
Downward sloping (inverted)



Flat



Hump shaped



The Expectations Hypothesis

- Assumptions
  - No transaction costs
  - Investors are risk-neutral
- Implication of risk-neutrality:
  Investors choose the maturity of their bonds to maximize holding period returns

Main Insights

- All expected HPR are equal in equilibrium ⇒
  - The long term rate is a geometric average of current and expected future short rates

$$(1+y_{2,t})^2 \approx (1+y_{1,t})(1+E_t[y_{1,t+1}])$$

$$1+y_{2,t} \approx [(1+y_{1,t})(1+E_t[y_{1,t+1}])]^{1/2}$$

  - Typical shape of yield curve is flat
  - Short-term interest rates are more volatile than long-term interest rates
- What does an upward-sloping term structure ($y_2 > y_1$) imply about the expected future short-term interest rate?

Link With Forward Rates

- Recall the *definition* of the forward rate

$$(1+y_2)^2 = (1+y_1)(1+f_2)$$

- Compare with the *prediction* of the EH

$$(1+y_{2,t})^2 \approx (1+y_{1,t})(1+E_t[y_{1,t+1}])$$

- Therefore, under the EH, the forward rate equals the expected future 1-year interest rate

$$f_2 \approx E_t[y_{1,t+1}]$$

Historical Yield Curves

- 10/2011: y_2=0.30%, y_5=1.14%, y_{10}=2.32%
  - slope YC = +333 bp (30yr – 1mth)
  - Under the expectations hypothesis:
    avg short rate 3-5 = 1.70%
    avg short rate 6-10 = 3.51%
- 04/1980: slope YC = -190 bp

Empirically, the slope of the yield curve is a good predictor of the GDP growth rate.

Is the EH Adequate?

- Problem with the EH: yield curve under the EH is flat on average while in the data it is upward sloping 90% of the time
- Source of problem: risk-neutrality assumption in EH
- But the short rate next period is not known, it is **risky**!

Liquidity Preference Theory

- When investors are **risk averse**, they care not only about the expected short rate but also about its volatility
- Investors in long-term bonds want to be compensated
  - For "tying up" money for a long time (inflation/purchasing power risk)
  - For facing price risk if they need to sell before maturity (interest rate risk)

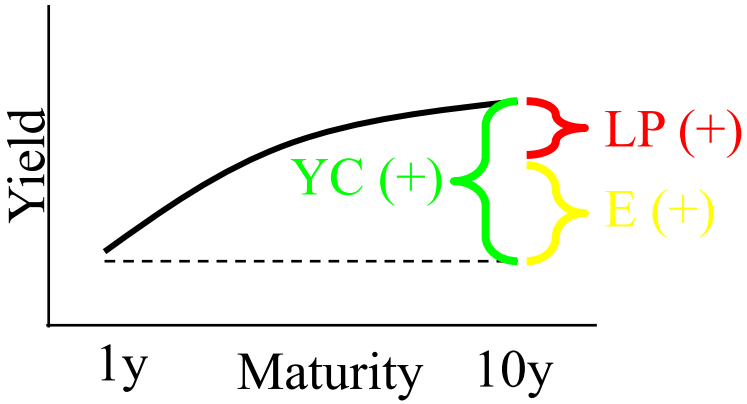
Liquidity Preference Theory

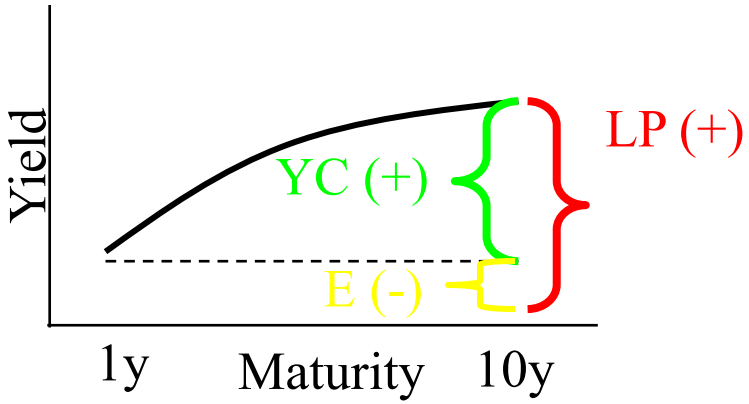
- The associated risk premium is denoted the liquidity premium (LP)

$$1 + y_{2,t} \approx \left[\left(1 + y_{1,t}\right)\left(1 + E_t\left[y_{1,t+1}\right]\right)\right]^{1/2} + LP$$

- Based on this theory
  - What is the typical shape of the term structure?
  - Is the forward rate still equal to the expected future short rate?

Examples





YC is the total yield spread
LP is yield spread due to risk (liquidity) premium
E is yield spread due to expected future changes in short rate

Conclusion

- Why does the term structure look like it does?
  - Expectations of future rates
  - Risk premiums
- These phenomena are both embedded in forward rates

Assignments

- Reading
  - BKM: Chapters 10.5, 11.1, 11.3
  - Problems: 11.1-11.2, 11.4, 11.6-11.10, 11.12, CFA 11.1-11.2, 11.10
- Assignments
  - Problem Set 5 due 14th November