# Session 20: Fixed Income III

Fall 2025

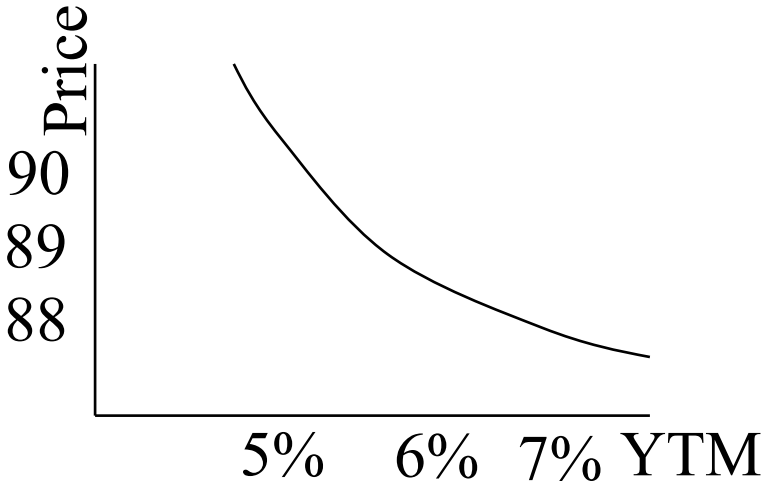
# Outline

- Interest rate sensitivity (of bond prices)
- Duration (convexity)
- Default risk

# Interest Rate Sensitivity

- First order effect: Bond prices and interest rates are negatively related
- Maturity matters: Prices of long-term bonds are more sensitive to interest rate changes than short-term bonds
- Convexity: An increase in a bond's YTM results in a smaller price decline than the price gain associated with a decrease of equal magnitude in the YTM

# An Example



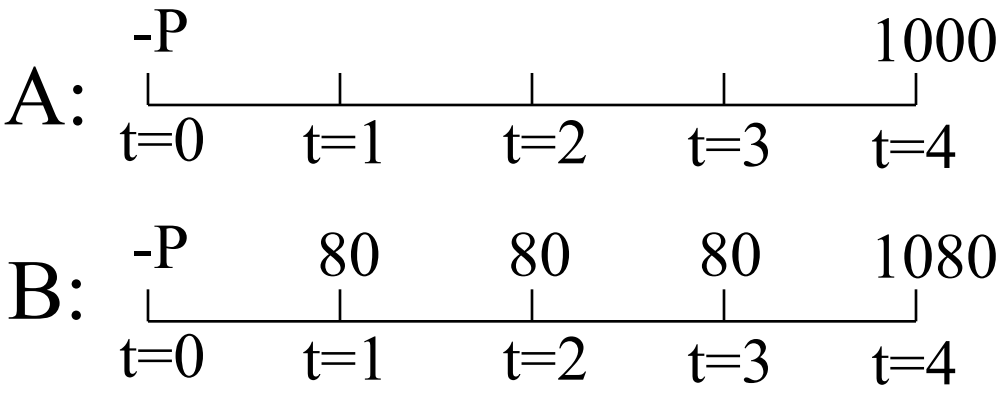

| YTM | 1-year | 2-year |
|---|---|---|
| 5% | 95.24 | 90.70 |
| 6% | 94.34 | 89.00 |
| 7% | 93.46 | 87.34 |

- Negative convex relation between YTM and price:
  - Price change 2-year ZCB 6%→5%: 90.70 - 89.00 = 1.70
  - Price change 2-year ZCB 6%→7%: 87.34 – 89.00 = -1.66
- Maturity matters:
  - % change 1y 6%→7%: (93.46 – 94.34)/94.34 = -0.93%
  - % change 2y 6%→7%: (87.34 – 89.00)/89.00 = -1.86%

# Duration: The Concept

Duration (or Macaulay duration)

- The sensitivity of the price of a bond to changes in the yield
- Can often be interpreted as the "average" time you have to wait for your payments



# The Derivation

The price sensitivity is linked to the average time you have to wait for your payments, provided the weights are the contributions to the price of the bond:

$$P = \frac{CF_1}{(1+y)^1} + \frac{CF_2}{(1+y)^2} + \ldots + \frac{CF_T}{(1+y)^T}$$

$$\frac{dP}{dy} = -1\frac{CF_1}{(1+y)^2} - 2\frac{CF_2}{(1+y)^3} - \ldots - T\frac{CF_T}{(1+y)^{T+1}}$$

$$-\frac{dP}{dy}\frac{1+y}{P} = 1\frac{CF_1}{P(1+y)^1} + 2\frac{CF_2}{P(1+y)^2} + \ldots + T\frac{CF_T}{P(1+y)^T}$$

# Summary

- The duration ($D$) of a bond is defined as minus the **elasticity** of its price ($P$) with respect to (1 plus) its YTM ($y$):

$$D = -\frac{dP}{dy}\frac{1+y}{P} = \sum_{1}^{T} w_t\, t \quad \text{where} \quad w_t = \left(\frac{CF_t}{(1+y)^t}\right) \Big/ P = PV(CF_t)/P$$

- For fixed cash flows, duration is equal to the average of the cash-flow times, weighted by their contribution to the present value of the bond
- The price response to a yield change is therefore

$$\frac{\Delta P}{P} \cong -\underbrace{\frac{D}{1+y}}_{\substack{\text{modified} \\ \text{duration}}} \Delta y$$

# An Example

What is the duration of a 3-year coupon bond with a face value of $1000, a coupon rate of 8%, and a YTM of 10%?

# An Example cont'd

- If the YTM changes to 10.1%, what would be the (relative) change in price?

- Does the duration formula imply that long-term bonds are more risky (bigger price changes) than short-term bonds?

# Useful Facts

- What is the duration of zero-coupon bond?
- What must be true for the duration of a coupon bond?
- What happens to the duration of a coupon bond if (all else equal) the coupon rate increases?
- What happens to the duration of the bond if (all else equal) the YTM increases?

# Useful Facts cont'd

- Factors affecting duration
  - Maturity (+)
  - Coupon rate (-)
  - YTM (-)
- The duration of a portfolio is the weighted average of the durations of the constituents: $D_p = \sum_i w_i D_i$
- What is the duration of a perpetuity?

# Convexity

- The sensitivity of price with respect to yield is approximated by a linear function when using duration
- The relation is really non-linear (convex)
  - When yields decline, the price increase in the bond is underestimated by the simple duration formula
  - When yields increase, the price decline in the bond is overestimated by the simple duration formula
  - A convexity term corrects the problem

# Default Risk

- Not all bonds are risk-free, i.e., there is some chance the actual payments will not equal the promised payments
- The amount of default (credit) risk is summarized by the bond rating

Moody's: Aaa Aa A Baa Ba B Caa Ca C D

S&P: AAA AA A BBB BB B CCC CC C D

Investment grade

Speculative grade
High yield
Junk bonds

# Default Risk Premiums

- Risky bonds offer higher yields than otherwise comparable risk-free to securities
- This yield premium is made up of two components
  - Compensation for the difference between promised cash flows and expected cash flows
  - A risk premium, i.e., a higher expected return to compensate for the risk (think SML!)
- The YTM on risky bonds is NOT the expected return

# Conclusion

- Bonds are exposed to 2 types of risk
  - Interest rate risk (duration)
  - Default risk (rating)
- In general, investors require compensation for these risks in the form of higher yields

# Assignments

- Reading
  - BKM: Chapters 11.2, 11.4
  - Problems: 11.11, 11.15-11.17, 11.20
- Assignments
  - Problem Set 5 due the 14th November