Session 23: Options II

Fall 2025

Outline

- Valuation of an option prior to expiration
  - Intrinsic value
  - Minimum value
  - Put-call parity
- Binomial valuation

Intrinsic Value of a Call Option

- The **intrinsic value** is
  - ✓ The value of the right to exercise *now*
- What is the intrinsic value of an out-of-the-money call?
- What is the intrinsic value of an in-the-money call?
- Which is greater, the intrinsic value or the option price?
- The difference between the option price and the intrinsic value is called the **time value** of the option. (Note: this is not related to time value of money.)

Minimum Value of Call

- The **adjusted intrinsic value** is the present value of the payoff when forced to decide today whether to exercise at T or not
  - Present value if exercised: $PV(S_T - X) = S_0 - Xe^{-rT}$
  - Present value if not exercised: $PV(0) = 0$
  - Adjusted intrinsic value: $\max(0, S_0 - Xe^{-rT})$
- With a call option, you are not forced to decide today whether or not to exercise at T, so the adjusted intrinsic value is a **<u>lower bound</u>** on the call option value:

$$C_0 \geq \max\left(0, S_0 - Xe^{-rT}\right)$$

Adjusted Intrinsic Value

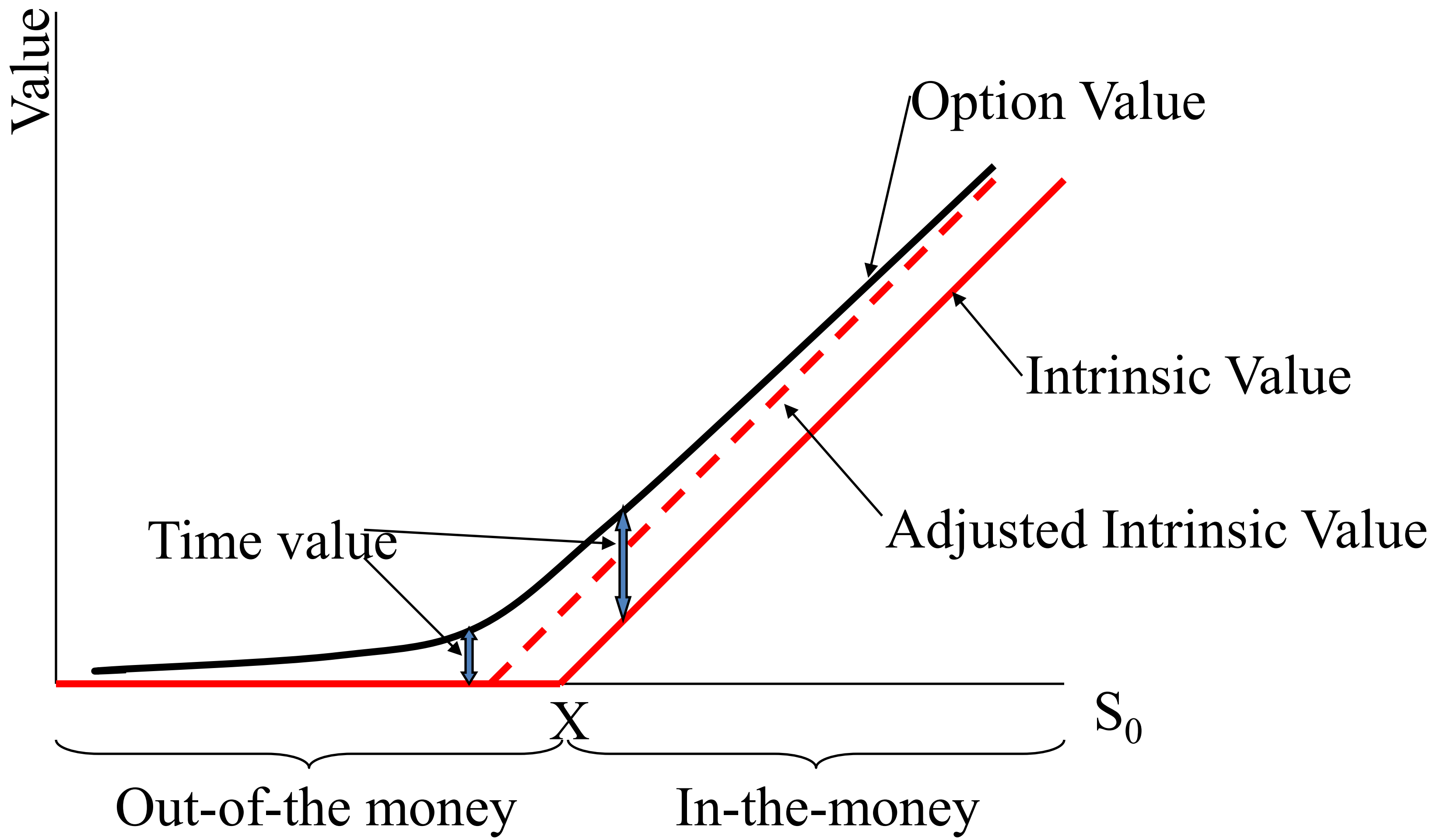
- The arbitrage proof: $C_0 \geq \max\left(0, S_0 - Xe^{-rT}\right)$

Imagine $C_0 < S_0 - Xe^{-rT}$

	0	T
Buy call	$-C_0$	$S_T - X$
Sell stock	S_0	$-S_T$
Lend	$-Xe^{-rT}$	X
	$(S_0 - Xe^{-rT}) - C_0$	0

An example: $S = 105$, $X = 100$, $r = 5\%$, $T = 1$
Intrinsic value:
Adjusted intrinsic value:

Minimum Value of Call



Put-Call Parity

(Non-dividend paying stocks)

You can create a “synthetic” stock by buying a portfolio of European options and risk-free assets:

- Buy a call with strike X and expiration date T, sell a put with same X and T, and buy a zero-coupon bond with face X and maturity T
- Payoff (at time T)

$$\max(0, S_T - X) - \max(0, X - S_T) + X = S_T$$

Put-Call Parity cont'd

By the law of one price (no arbitrage)

$$C_0 - P_0 + Xe^{-rT} = S_0 \quad \Rightarrow \quad C_0 + Xe^{-rT} = S_0 + P_0$$

$$\Rightarrow \quad C_0 = S_0 + P_0 - Xe^{-rT} \quad \Rightarrow \ldots$$

An example: S = 105, X = 100, r = 0, T = 1

C = 10 → P =

P = 8 → C =

American Options

Exercisable at or *before* expiration

- American call option on a non-dividend paying stock: Is it ever optimal to exercise before expiration?

- American call option on a dividend paying stock: Is it ever optimal to exercise before expiration?

American Options

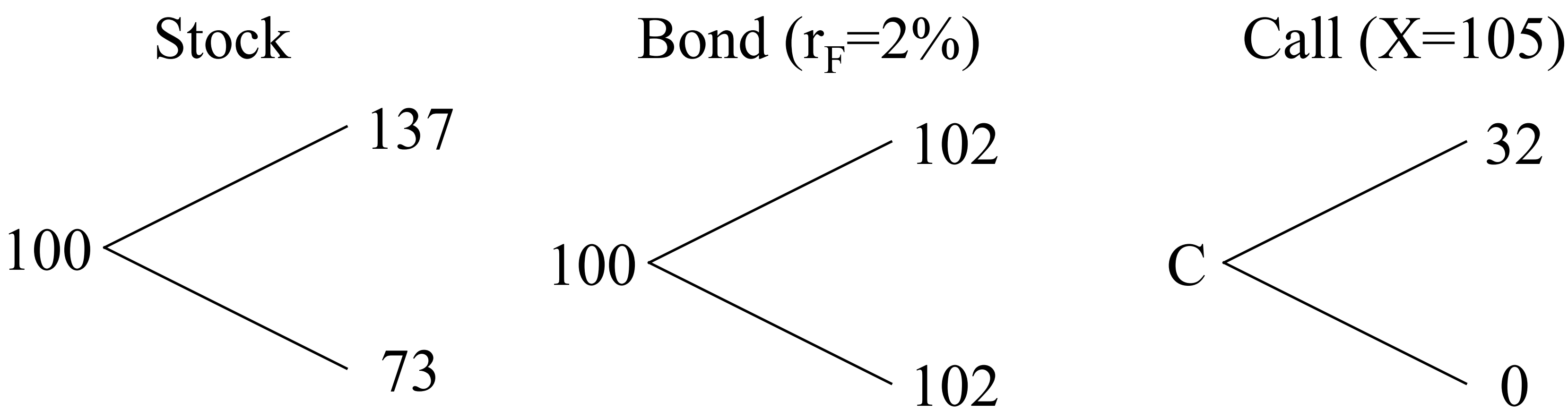
- Reasons *not to* exercise early
  - For calls—the time value of money on the payment of the exercise price
  - For calls and puts—option value
- Reasons *to* exercise early
  - For puts—the time value of money on the receipt of the exercise price
  - For calls—receipt of dividends
- In the absence of dividends, it is never optimal to exercise *calls* early

What is an Option Worth?

Binomial Valuation

Consider a world in which the stock can take on only 2 possible values at the expiration date of the option. In this world, the option payoff will also have 2 possible values. This payoff can be replicated by a portfolio of stock and risk-free bonds. Consequently, the value of the option must be the value of the replicating portfolio.

Payoffs



1-year call option, S=100, X=105, r_F=2% (annual)

Can the call option payoffs be replicated?

Replicating Strategy

Buy ½ share of stock, borrow \$35.78 (at the risk-free rate).

Payoff

(½)137 - (1.02) 35.78 = 32

Cost

(1/2)100 - 35.78 = 14.22

(½)73 - (1.02) 35.78 = 0

The value of the option is \$14.22!

Solving for the Replicating Strategy

The call option is equivalent to a levered position in the stock (i.e., a position in the stock financed by borrowing).

$$137\,H - 1.02\,B = 32$$

$$73\,H - 1.02\,B = 0$$

$\Rightarrow$

$$H = \frac{32-0}{137-73} = \frac{1}{2} = \frac{C_u - C_d}{S_u - S_d}$$

$$B = \frac{137(0.5)-32}{1.02} = 35.78 = \frac{S_u H - C_u}{1+r_f}$$

$$C = 0.5(100) - 35.78 = 14.22 = HS - B$$

Binomial Pricing

- The probabilities of the 2 states are apparently irrelevant. Why?
- Why not use DCF to price the option?
- The idea of binomial valuation via replication is incredibly general. If you can write down a binomial asset value tree, then any (derivative) asset whose payoffs can be written on this tree can be valued by replicating the payoffs using the original asset and a risk-free, zero-coupon bond.

Conclusion

- Option pricing bounds
- Static replication: put-call parity
- Dynamic(?) replication: pricing!

Assignments

- Reading
  - BKM: Chapters 16.3-16.5
  - Problems: 16.2-16.6, 16.8, 16.10-16.12, 16.14-16.17, 16.19-16.22, 16.27
- Assignments
  - Problem Set 6 due 1st December