# Session 25: Futures

Fall 2025

# Outline

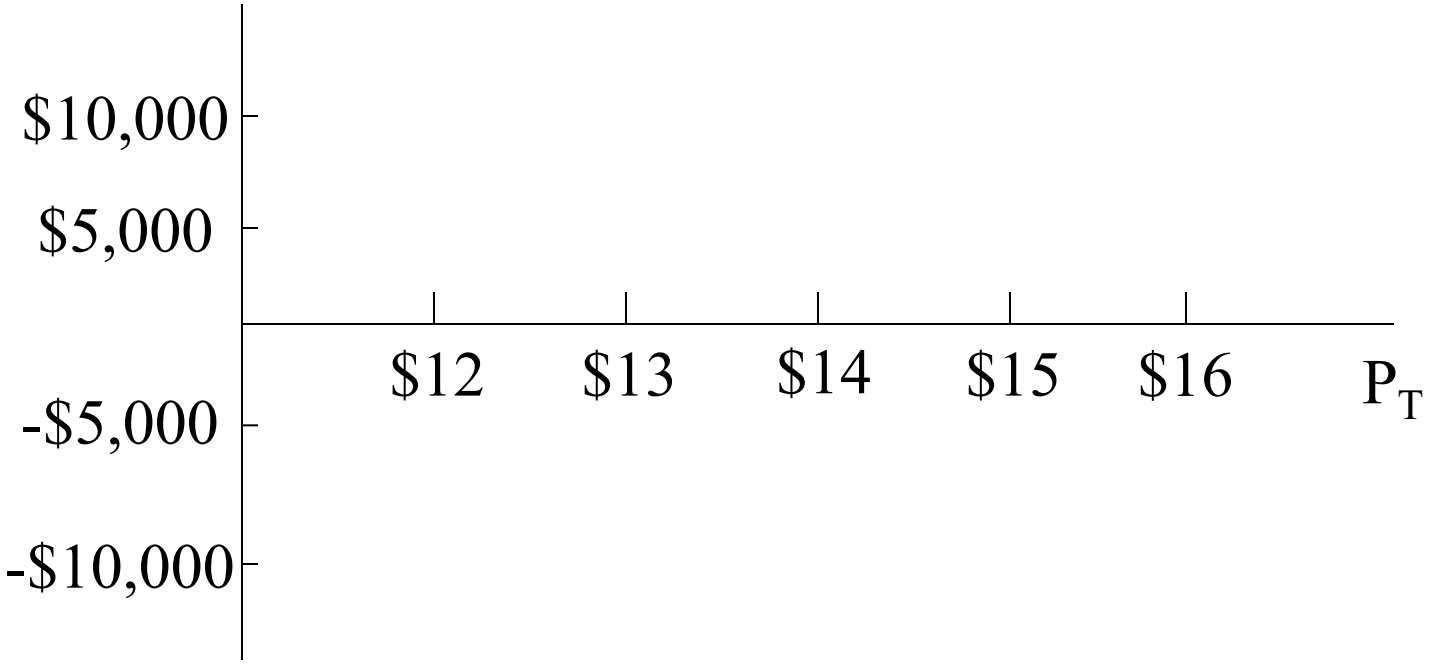
- Contractual agreements (futures vs. forwards)
- Payoff
- Participants
- Financial futures pricing
  - Stock index
  - Currency
- Commodity futures pricing

# Contractual Arrangements

- Future/forward: deferred delivery contract of underlying assets
- Buyer (seller) has *obligation* to take (make) delivery on settlement date at agreed upon price
- Futures
  - Marked-to-market (daily cash settlement)
  - Often standardized contract
  - Exchange traded/clearing house
- Forwards
  - No cash transfer until maturity
  - Tailor made contract
  - OTC
- What are the advantages of futures?

# Payoff Forward

What is the payoff (long and short) of a forward contract for 5000 oz of silver, $14 per oz, to be delivered in 6 months?



# Participants

- Who are the people who buy and sell futures?
  - Hedgers
  - Speculators
  - Arbitrageurs and scalpers
- Hedgers combines position in cash/spot market (S) with futures position (F) to minimize risk from cash price fluctuations
- Hedge ratio: $H = \Delta S/\Delta F$

# Example: Hedging with Futures

- A jeweler has 1000 oz of gold
- A $\Delta S$ change in spot gold price leads to
  - A $1000(\Delta S)$ change in wealth
  - A $\Delta F$ change in futures price
- Hedging (futures position X ) means
  - $1000(\Delta S)+X(\Delta F) = 0$, hence $X = -1000(\Delta S/\Delta F)$
  - With hedge ratio $H = \Delta S/\Delta F$
- If spot and futures price move 1 to 1 ($H = 1$), he needs to sell futures on 1000 oz
- What if $H = 0.5$?

# Financial Futures: Stock Index

When are stock index cash and futures prices in proper alignment (assume no dividends)?

- Intuition: at time 0 (for a T-period contract)

$$\text{PV}(F_0) = \frac{F_0}{(1+r_f)^T} = \text{PV}(S_T) = S_0 \quad \Rightarrow \quad F_0 = S_0(1+r_f)^T$$

- Spot-futures parity is a no-arbitrage condition!

# Stock Index Futures: Example

- For a (known) dividend yield
- S&P500
  - S = 1200, r = 0.5%, d = 2.5%, T = 1
  - F = 1200(1+0.5%-2.5%) = 1176

$$F_0 = S_0(1 + r_f - d)^T$$

# Financial Futures: Currency

- When are the spot exchange rate ($S_0$ = \$AUS/\$US) and the forward exchange rate in alignment?
- There are two ways to get risk-free Australian dollars in the future:
  - Exchange US\$s for AUS\$s today, invest at the risk-free Australian rate $S_0(1+r_{\text{Aus}})^T$
  - Invest US\$s at the US risk-free rate and exchange US\$s for AUS\$s in the future at the forward rate $(1+r_{\text{US}})^T F_0$
  - By no-arbitrage (covered interest parity)

$$(1+r_{\text{US}})^T F_0 = S_0(1+r_{\text{Aus}})^T \quad \Rightarrow \quad F_0 = S_0\left(\frac{1+r_{\text{Aus}}}{1+r_{\text{US}}}\right)^T$$

# The Carry Trade

- Data
  - U.S. 1-year T-bill yield: 0.5%
  - Australian 1-year gov't bond yield: 3.5%
  - $AUS/$US exchange rate: 1.00
- How would you profit from the interest rate difference?
- What is the breakeven exchange rate 1 year from now?
- What kind of risk do you take?

# The Carry Trade cont'd

- How could you hedge the exchange rate risk?

- If you hedge the exchange rate risk, what is your profit from the carry trade?

# Commodity Futures

- Are cash and futures prices in proper alignment?
- There are two ways to get a commodity in the future
  - Borrow money to buy the commodity today at the spot price and pay back the loan in the future

    $S_0(1+r_f+s)^T$

    Physical storage cost: annual rate s
  - Buy the commodity in the futures market
  - By no-arbitrage

    $F_0 \le S_0(1+r_f+s)^T$

# Commodity Futures cont'd

More generally

$$F_0 = S_0(1 + r_f - c)^T$$

$c$ is the convenience yield, i.e., the net value of having the physical commodity relative to the futures contract (or the cost of borrowing the physical commodity to short it)

# Price Patterns

- *Contango*—the expected spot price is lower than the futures price, i.e., higher prices for more distant delivery
- *Backwardation*—the expected spot price is higher than the futures price, i.e., lower prices for more distant delivery

# Conclusion

- Futures (as with most derivatives) are priced off the underlying by no-arbitrage
- In commodity futures, the convenience yield may add an additional complication