Session 3: Time Value of Money

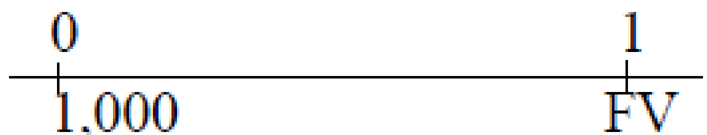
Foundations of Finance
Spring 2026

Outline

- Future value (FV), present value (PV), and yield (r)
- Examples
  - Single payment securities, e.g., zero coupon bonds
  - Multiple payment securities, e.g., annuities and perpetuities

Future Value: One Period

- What is the value of $1000 invested for one year at an annual rate of 5%?

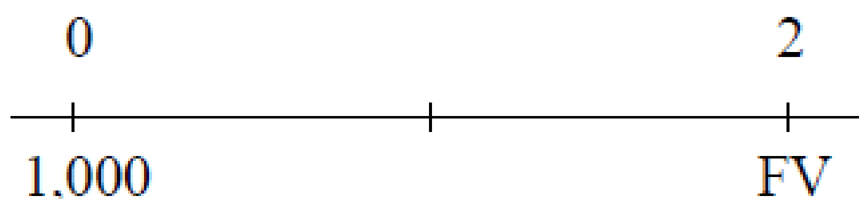


$$FV = PV(1+r) = 1000(1+5\%) = 1000(1.05) = 1050$$

- Principal plus (simple) interest

Future Value: Multiple Periods

- What is the value of $1000 invested for two years at an annual rate of 5%?



$$FV = PV(1+r)(1+r) = 1000(1.05)^2 = 1102.50$$
$$FV = 1000 + 2(5\%)1000 + 5\%(5\%)1000$$

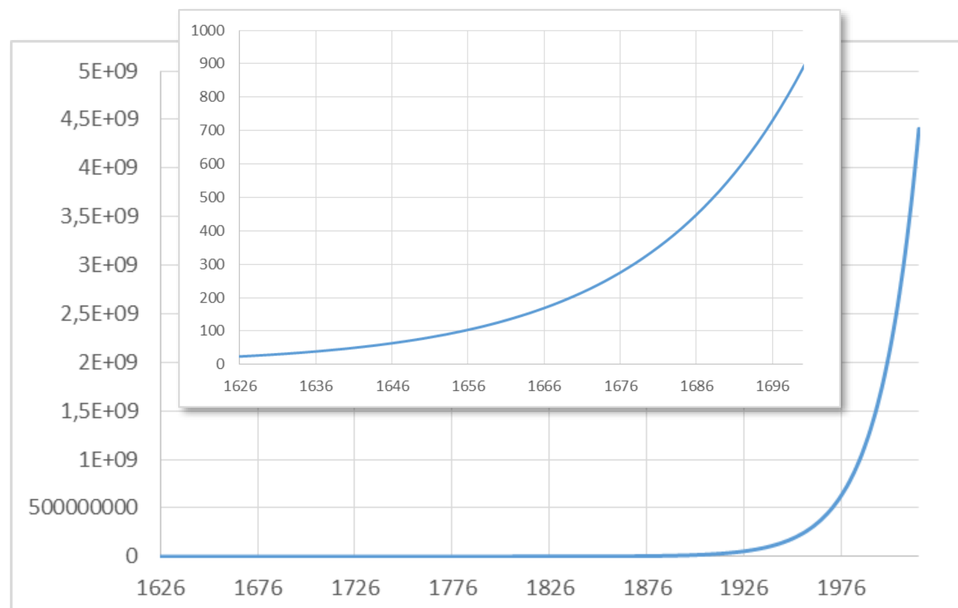
- Principal plus simple interest plus compound interest
- More generally: $FV = PV(1+r)^t$

Markets: New York Real Estate

- The Dutch West India Company dispatched the first permanent settlers to Manhattan Island in 1624. In 1626, the fledgling town's governor, Peter Minuit, bought Manhattan from the Canarsie tribe for $24 worth of beads and trinkets. Locals sometimes cite this transaction as one of the last real estate bargains in New York.
- How big of a bargain was it at a 5% interest rate?

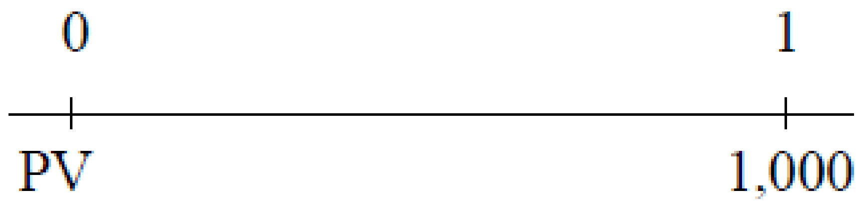
Assume 5% interest rate, the future value of
$24 is 24(1.05)^397 = $6.2 billion
(6,199,590,005)
A lot of money, but a good deal for the Dutch.

How much of this is simple interest?
24(0.05)(397)=$476,4 (The power of
compounding!!)



Present Value: One Period

- To receive $1000 in one year, how much should I invest today at a rate of 5%?

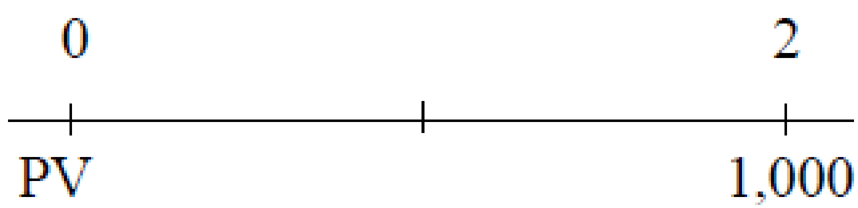


$$PV = FV/(1+r) = 1000/(1+5\%) = 1000/1.05 = 952.38$$

- This formula should look very familiar!

Present Value: Multiple Periods

- To receive \$1000 in two years, how much should I invest today at a rate of 5%?

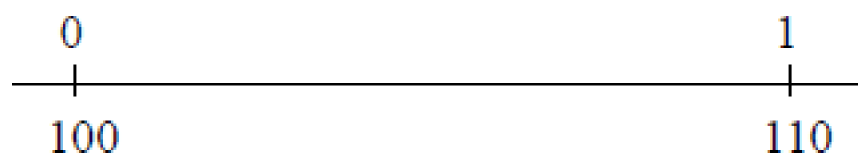


$$PV = [FV/(1+r)]/(1+r) = 1000/1.05^2 = 907.03$$

- In general: $PV = FV/(1+r)^t$

Discount Rate/Yield

- One period
  - If you invest \$100 and receive \$110 in 1 year, what rate are you earning?



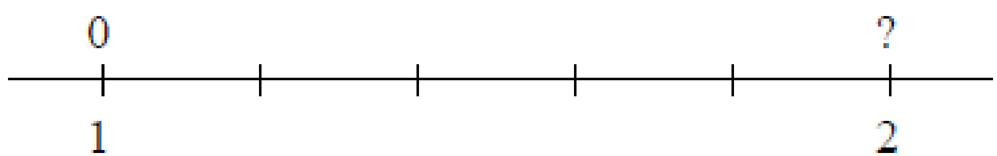
r = (FV/PV)-1 = (110/100)-1 = 0.1 = 10%

- Multiple periods
  - If an investment offers to double your money in 5 years, what rate are you earning?

$r = (FV/PV)^{1/t} - 1 = (2/1)^{1/5} - 1 = 14.87\%$

Time

- At a rate of 5% how long will it take to double your money?



t = log(FV/PV)/log(1+r)

t = log(2/1)/log(1.05) =14.21 years

Recap

- Given 3 you can always find the 4th

$$FV = PV(1+r)^t$$

$$PV = FV/(1+r)^t$$

$$r = (FV/PV)^{1/t}-1$$

$$t = \log(FV/PV)/\log(1+r)$$

Zero Coupon Bonds

- What is a zero coupon bond?

  A bond that pays no interest/coupon, only the face value at maturity.

- Where do zero coupon bonds come from?
  - Issued in primary markets (Treasury bills)
  - Stripping of coupon bonds (example of "financial engineering")

Zero Coupon Bond
F=100, r=5%, t=10
Price?

Pricing Zero Coupon Bonds

Price = PV = $F/(1+r)^t$

$F=100$, $r=5\%$, $t=10 \rightarrow PV=100/(1.05)^{10}=61.39$

- $t=5 \rightarrow$ PV=

  $t=1 \rightarrow$ PV=

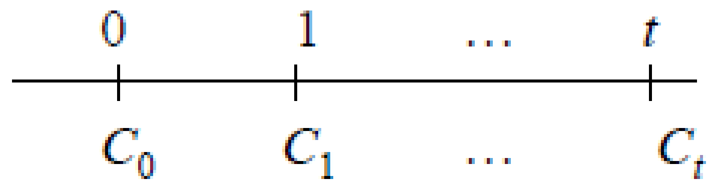
  Pull to par

- $r = 4\% \rightarrow$ PV =

  Lower interest rates → higher bond prices

Multiple Payments

- Present value of a series of cash flows $C_0, C_1, \ldots, C_t$



- Future value of the same cash flow stream

$$PV = C_0 + C_1 \frac{1}{(1+r)} + \ldots + C_t \frac{1}{(1+r)^t}$$

$$FV_t = C_0(1+r)^t + C_1(1+r)^{t-1} + \ldots + C_t$$

Multiple Payments: An Example

- What is the PV of a $1,000 par, 2-year, 5% coupon bond (annual payments) at a 6% interest rate?

0	1	2
PV	50	1,000+50

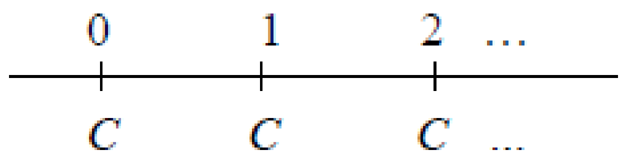
PV=50/1.06+1050/1.06^2=47.17+934.50=981.67

This is a discount bond (price<par)! Why?

- What is the present value at 5%?

Perpetuities

- Definition: Pays a fixed cash flow, C, every period, forever (starting at time 1)



- Pricing: PV = *C/r*

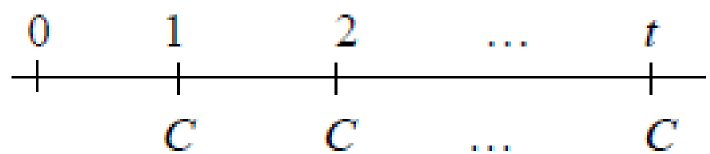
$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} \ldots + \frac{C}{(1+r)^t} + \ldots$$

Perpetuity: Example

- Suppose that maintenance of your grave costs $100 every year, forever
- The interest rate is 5% per year
- How much money should you leave your trustee?

Annuities

Definition: Pays a fixed cash flow, C, for t periods (starting at time 1)



$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} \dots + \frac{C}{(1+r)^t}$$

$$= C\,\frac{\left[1 - \dfrac{1}{(1+r)^t}\right]}{r} = C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$$

Annuity: Example

What car can you afford?

- You have no large amount of cash
- You can afford \$632 per month
- You can borrow at an interest rate of 1% per month
- You want to have paid the loan in full in 48 months

$$PV = C\left[\frac{1}{r} - \frac{1}{r(1+r)^t}\right]$$

$$= 632\left[\frac{1}{0.01} - \frac{1}{0.01(1+0.01)^{48}}\right] = 24{,}000$$

Interest Rates and Prices

- For bonds?

  Inverse relationship between price and interest rate. You give me the interest rate and I give you the price or vice versa

- For perpetuities?

  Inversely proportional. Double interest => half the price. This is unique to perpetuities

- For stocks?

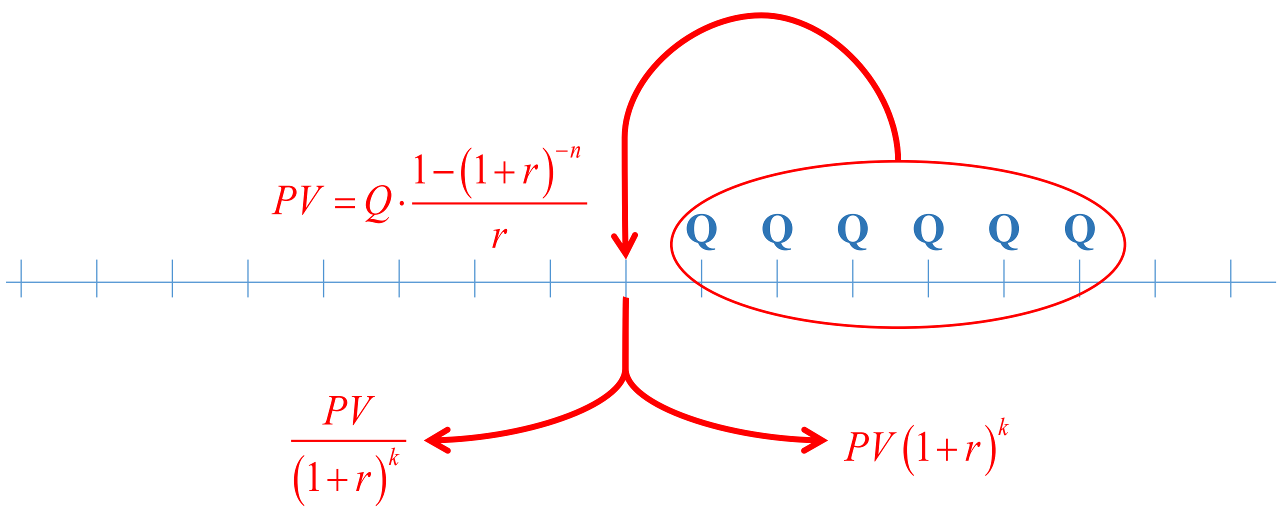
  inverse relation between price and discount rate (holding expected cash flows fixed!)

Conclusion

Time value of money is easy:

There is only one formula!

(perhaps 2)



Assignments

- Reading
  - RWJ: Chapter 5.3, 8.1, 8.4
    Handout 01 - Numerical Example - Valuating a zero coupon bond
    Handout 02 - Annuities and Perpetuities
    Handout 03 - Continuous Compounding

- Problems: 5.12-5.15, 5.43, 8.5-8.7, 8.25

- Assignments
  - Problem Set 1 due 2nd February