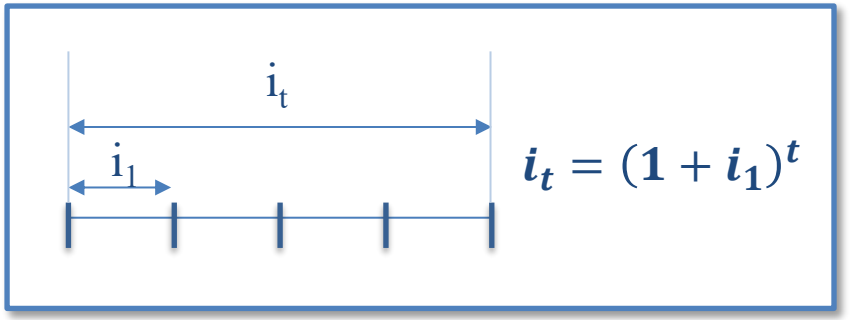
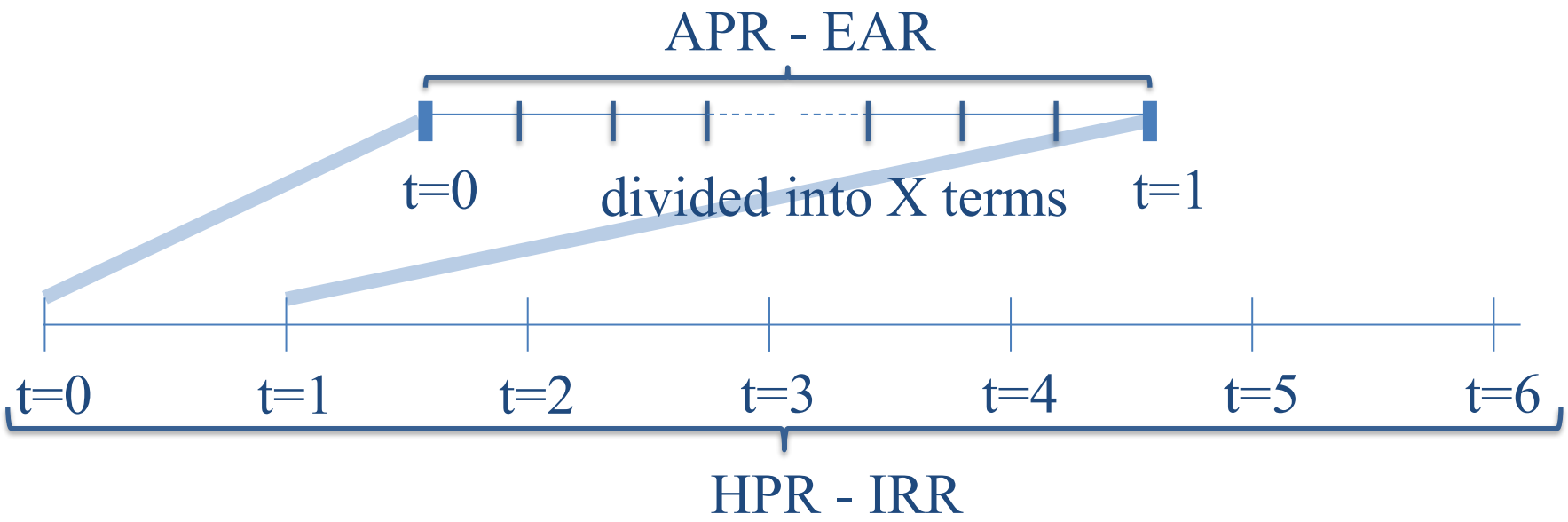
Session 4: Performance Measurement

Spring 2026

Outline

- Compounding
  - Annual percentage rate (APR)
  - Effective annual rate (EAR)
- Multi-period returns
  - Holding period return (HPR)
  - Internal rate of return (IRR)





APR

- Lenders are required by law to report the annual percentage rate (APR)
  APR = (rate per period)(# of periods per year)
- For example, a loan that charges 1% per month:
  APR = 1%(12) = 12%
- Is 12% per year the "real" cost?

APR vs. EAR

- Effective annual rate (EAR).
  If interest is compounded *m* times a year:

$$\text{EAR} = \left(1 + \text{APR}/m\right)^m - 1$$

- Which loan is cheaper?
  - 12%, compounded annually
  - 12%, compounded monthly

Continuous Compounding

- Consider increasingly frequent compounding: annually, quarterly, daily, every second,…
- When compounding happens "all the time," it is called continuous compounding
- What happens to the EAR?

$$\text{EAR} = \exp(\text{A}PR) - 1 = e^{\text{APR}} - 1$$

- Future value and present value

$$FV = PV\, e^{rt} \qquad PV = FV\, e^{-rt}$$

$$\boxed{\frac{1}{(1+i)^t} \approx e^{-t \cdot i}}$$

Recap

$$\text{EAR} = \left(1 + \text{APR}/m\right)^m - 1$$

$$\text{APR} = [(1 + \text{EAR})^{1/m} - 1]m$$

$$\text{EAR} = e^{\text{APR}} - 1$$

$$\text{APR} = \ln(1 + \text{EAR})$$

How do you make a loan seem cheaper?

https://www.contante.es/terminos/

Holding Period Return

- The holding period return (HPR) is the rate (return) that your initial investment earned in order to generate the final value of the investment
- The annual holding period return (ann. HPR) is the corresponding rate per year (annual return)

HPR: The Formula

- Example
  - At time 0, you buy an investment for V_0 (PV)
  - You re-invest all intermediate cash flows until date t
  - At time t, you sell the investment *and the re-invested cash-flows* for a total price of V_t (FV)
- The annual holding period return (ann. HPR) is the solution to:

$$V_0(1+\text{ann.HPR})^t = V_t$$

$$\text{ann.HPR} = \left(\frac{V_t}{V_0}\right)^{1/t} - 1$$

HPR: Stock

- Holding period return:

$$\text{HPR} = \frac{\text{ending price} + \text{ending cash dividend}}{\text{beginning price}} - 1$$

- Annual holding period return for a holding period of *t* years:

$$\text{ann. HPR} = (1 + \text{HPR})^{1/t} - 1$$

$$= \left(\frac{\text{ending price} + \text{ending cash dividend}}{\text{beginning price}} \right)^{1/t} - 1$$

Stock Example

- You bought Coca-Cola shares for $42.39 and sold them 6 months later for $44.30. Suppose there was no dividend payment in these 6 months.
  What are the HPR and ann. HPR?

- You bought KO shares for $39.63 two years ago and sold them for $42.37. Assume that the only dividend of $1.12 is paid at the end of year 2.
  What are the HPR and ann. HPR?

HPR: Zero-Coupon Bond

- F = \$1000, PV = \$435, t = 10 years
- $r = (1000/435)^{(1/10)}-1 = 0.0868 = 8.68\%$
  This r is the YTM but also the ann. HPR if you hold the bond until maturity
- What is the ann. HPR if you sell early?
  After 1 year for \$472.758?

  After 1 year for \$480?

  After 1 year for \$460?

Internal Rate of Return

IRR

- Average return
- Return if one can re-invest cash-flows at this rate
- Rate that makes
  initial price = present value of future cash flows

$$P_0 = PV = \sum_{t=1}^{\infty} \frac{C_t}{(1+IRR)^t}$$

IRR: Zero-Coupon Bond

- F = \$1000, PV = \$435, t = 10 years

$$P_0 = PV = \sum_{t=1}^{\infty} \frac{C_t}{(1+IRR)^t}$$

$$PV = \frac{F}{(1+IRR)^t} \Rightarrow IRR = \left(\frac{F}{PV}\right)^{1/t} - 1$$

- IRR = $(1000/435)^{(1/10)}$-1 = 0.0868 = 8.68%
- IRR=YTM=ann. HPR

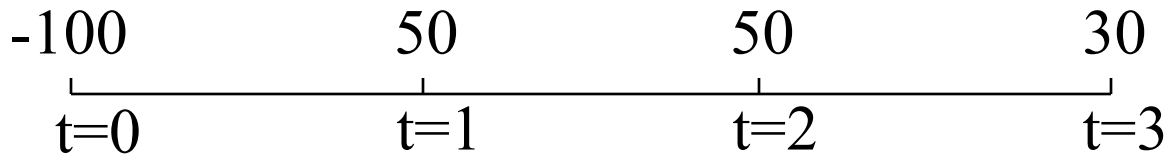
IRR: Dividend-Paying Stocks

- Coca Cola shares
  - Bought 1 share 2 years ago for $39.63
  - Earned dividend of $1.12 at end of each year
  - Sell the share for $44.30
- What is the IRR?

- What is the ann. HPR if you can reinvest the dividend at the IRR?

IRR: Project Valuation

- Pfizer wants to compute IRR on a potential project
- Business plan projects following cash-flows:



- Solve:

$$-100+\frac{50}{(1+IRR)}+\frac{50}{(1+IRR)^2}+\frac{30}{(1+IRR)^3}=0$$

- Answer: IRR=15.655%

Conclusion

There are a number of related (in some cases identical) ways of thinking about the return on (or the cost of) an investment:

- EAR
- ann. HPR
- IRR

Assignments

- Reading
  - BKM: Chapter 5.1-5.3
    Handouts: Statistics Review
    Geometric vs. Arithmetic Averages
  - Problems: 5.5, 5.7
- Assignments
  - Problem Set 1 due next Monday!