Session 7: Portfolio Theory III

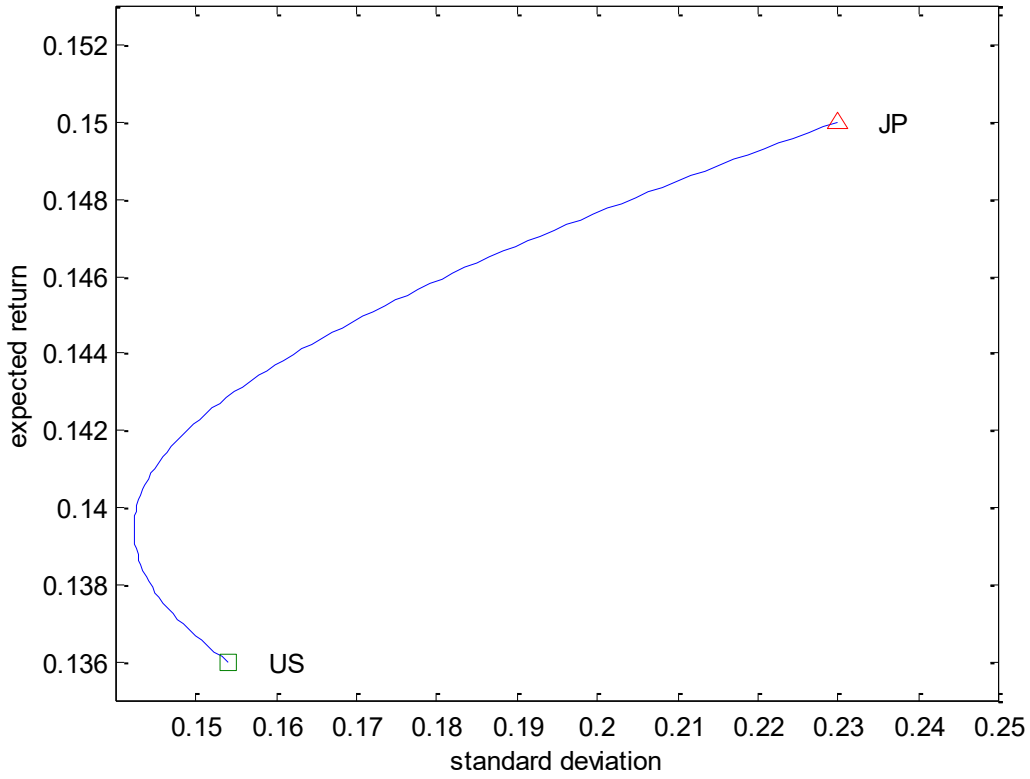
Spring 2026

Outline

- Risk-return tradeoff
- Indifference curves
- Optimal portfolio choice

Optimal Portfolio Choice

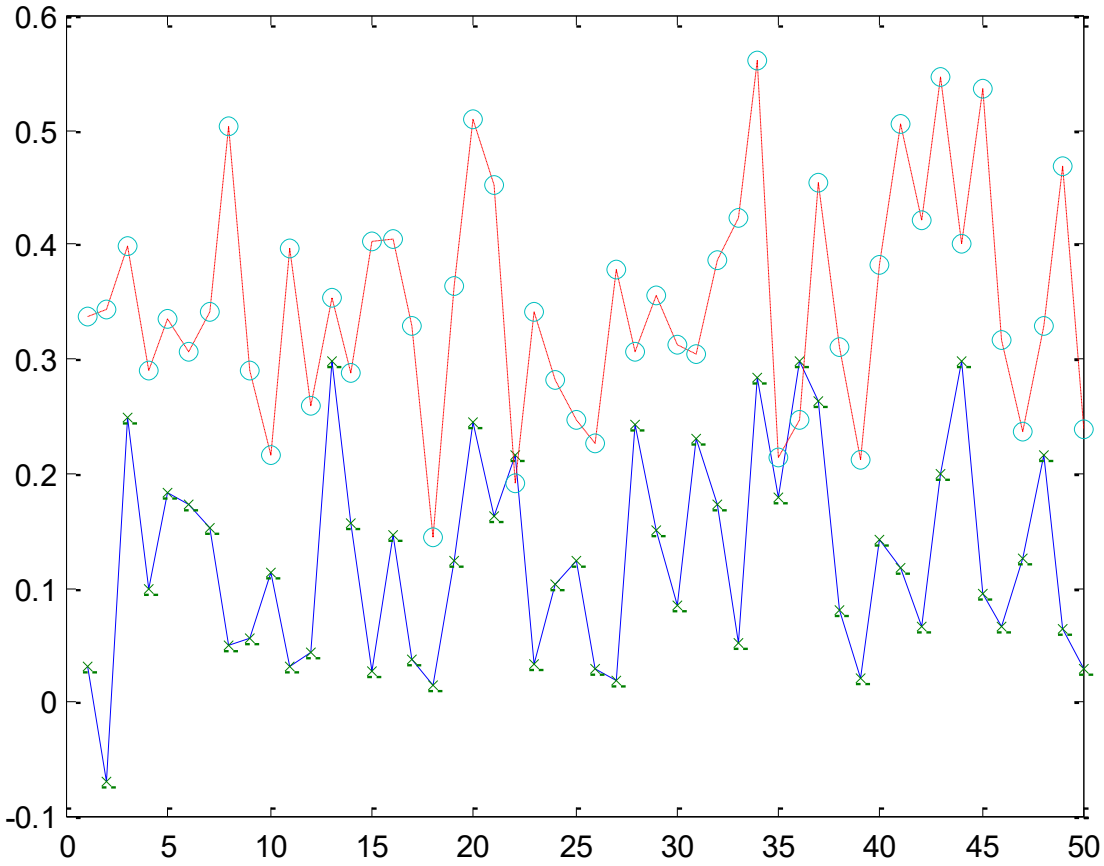
- Any (mean-variance) investor should choose an *efficient* portfolio to benefit from diversification
- Which efficient portfolio is *optimal* depends on the investor's preferences, in particular her risk aversion



Risk-Return Tradeoff

Which do you prefer?

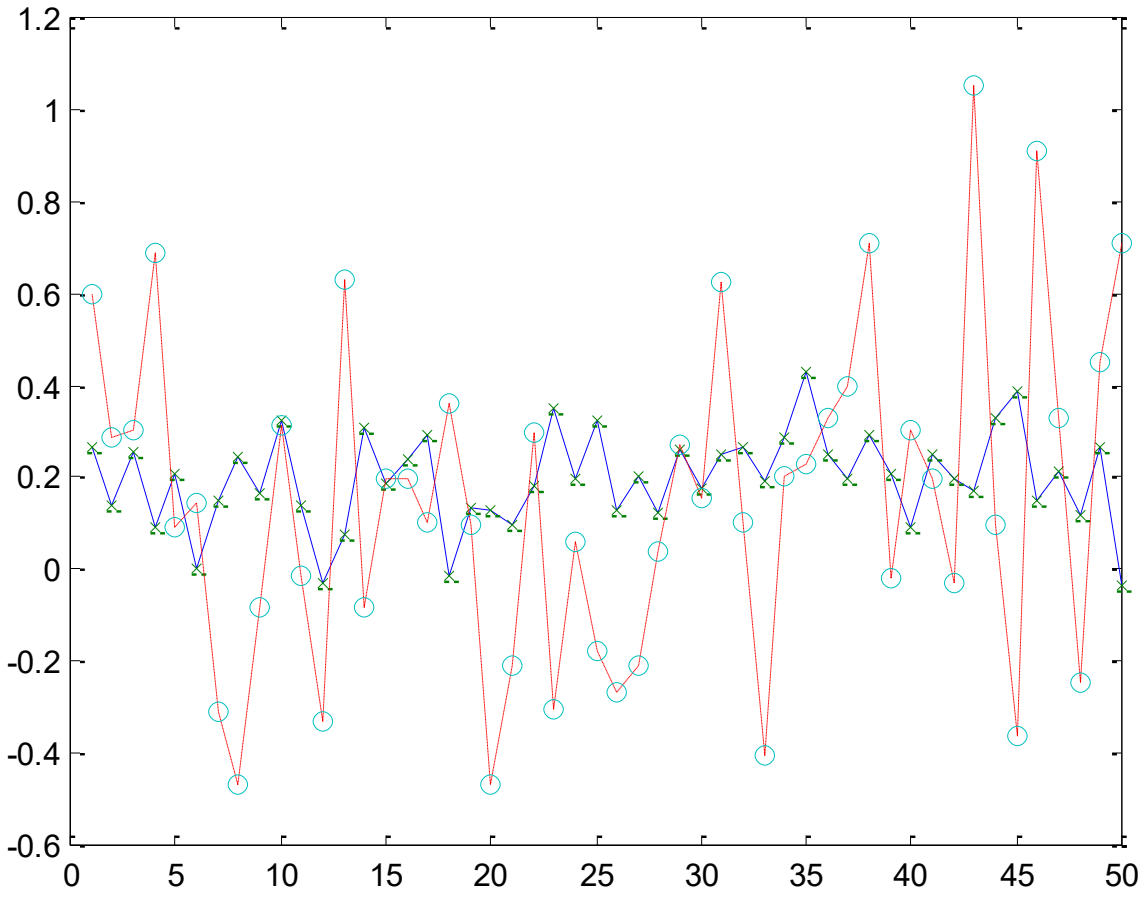
Mean	15%	35%
SD	10%	10%



Risk-Return Tradeoff

Which do you prefer?

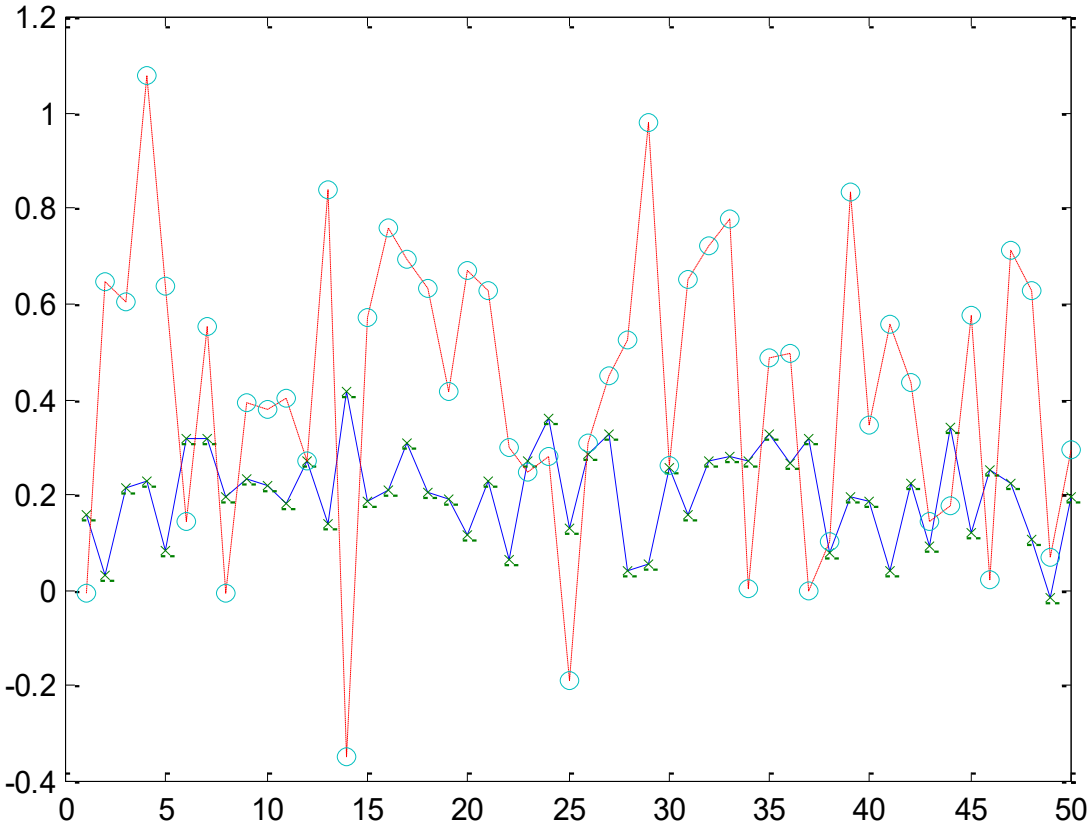
Mean	20%	20%
SD	10%	40%



Risk-Return Tradeoff

Which do you prefer?

		
Mean	20%	40%
SD	10%	40%

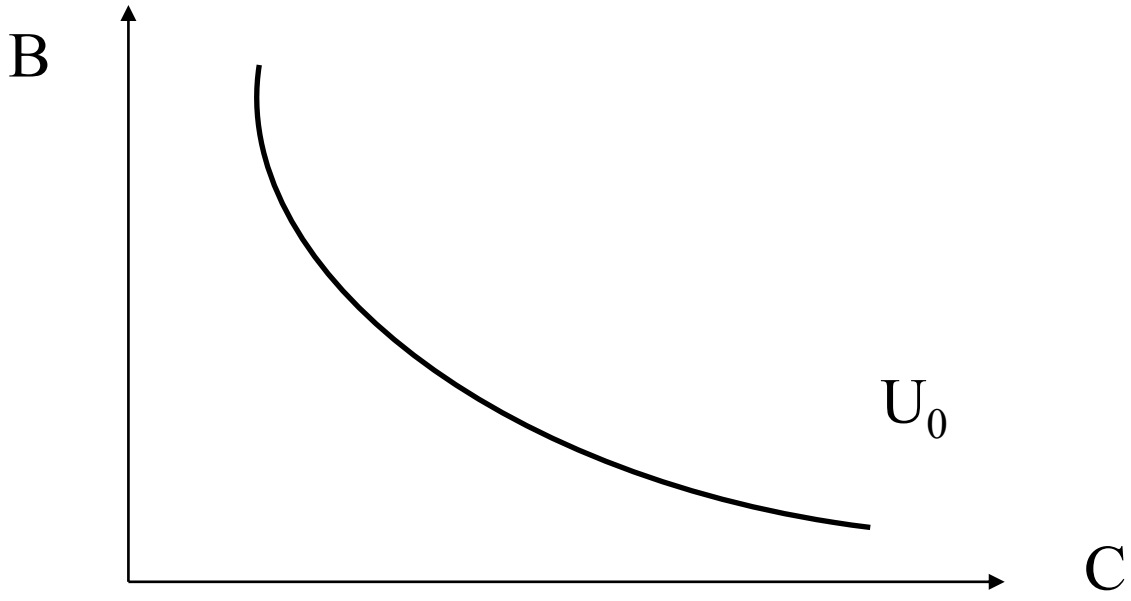


Risk-Return Tradeoff

- Recall two of the finance axioms:
  - Investors prefer more to less
  - Investors are risk averse
- This means that investors prefer an investment
  - With a higher expected return, $E(R_i)$
  - With a lower variance or standard deviation, σ_i
- Investors optimally **trade off** risk and return in order to maximize their expected utility

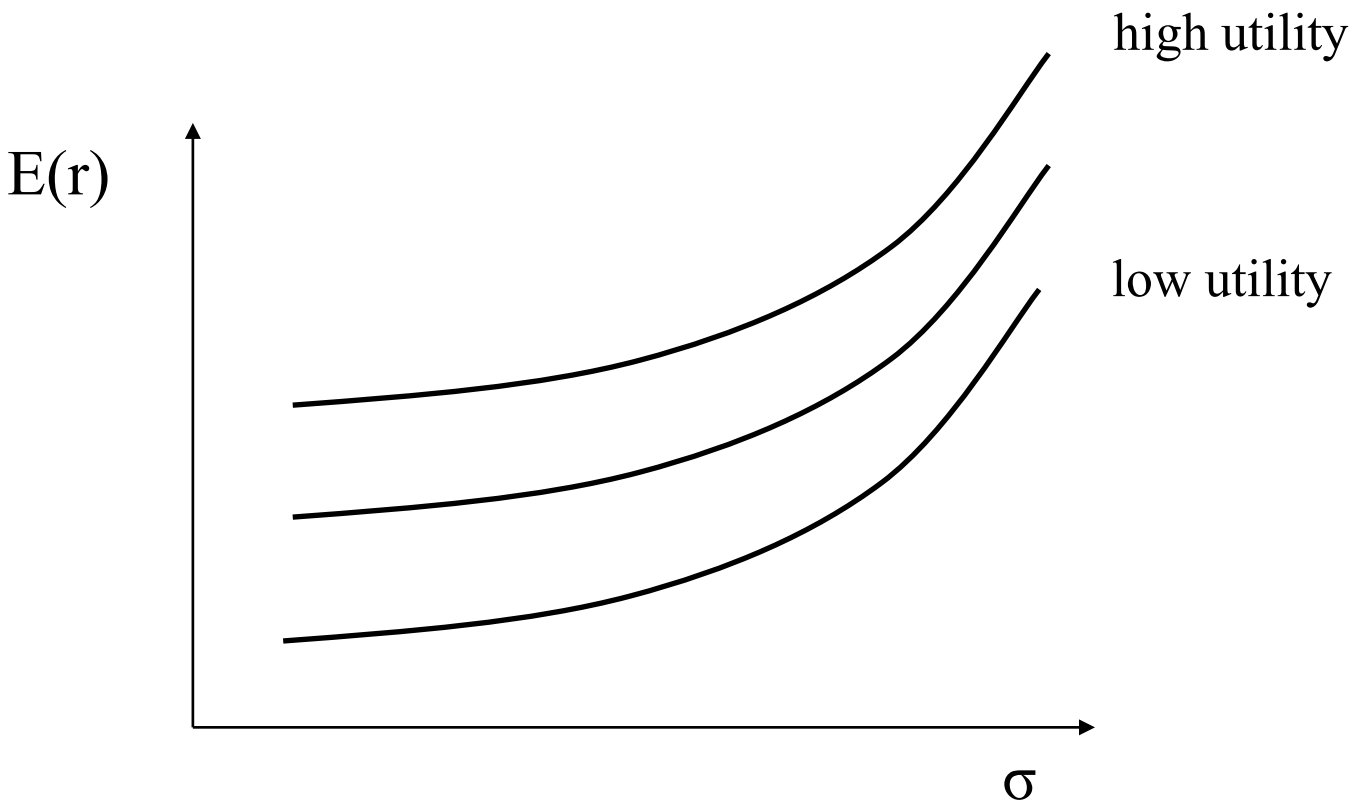
Indifference Curves: Review

- A person consumes 2 goods: beer (B) and chocolate (C)
- An indifference curve gives all the combinations of B and C that give the same utility level $U_0 = U(B,C)$
- People like to be on the highest possible indifference curve



Indifference Curves in Finance

Indifference curve: A set of $[E(r_p), \sigma_p]$ combinations that gives an investor the same expected utility



Utility Functions

- One utility function that is easy to work with is **mean-variance** utility:

$$U(r_p) = E(r_p) - 0.5A\,\mathrm{var}(r_p)$$

- A portfolio with higher utility score U has a more attractive risk-return profile
- Parameter $A>0$ measures **risk aversion**
- If $A=0$, investor is risk-neutral (does not care about risk, only about expected return)

Which is the "Better" Portfolio?

- Investor 1 has A=2, investor 2 has A=5
- Consider the following 2 investments:

Asset	Expected return	Standard Deviation
Low risk stock L	7%	5%
High risk stock H	13%	20%

- Calculate the utility scores:

Investor	Investment L	Investment H
Investor 1 (A=2)	U =	U =
Investor 2 (A=5)	U =	U =

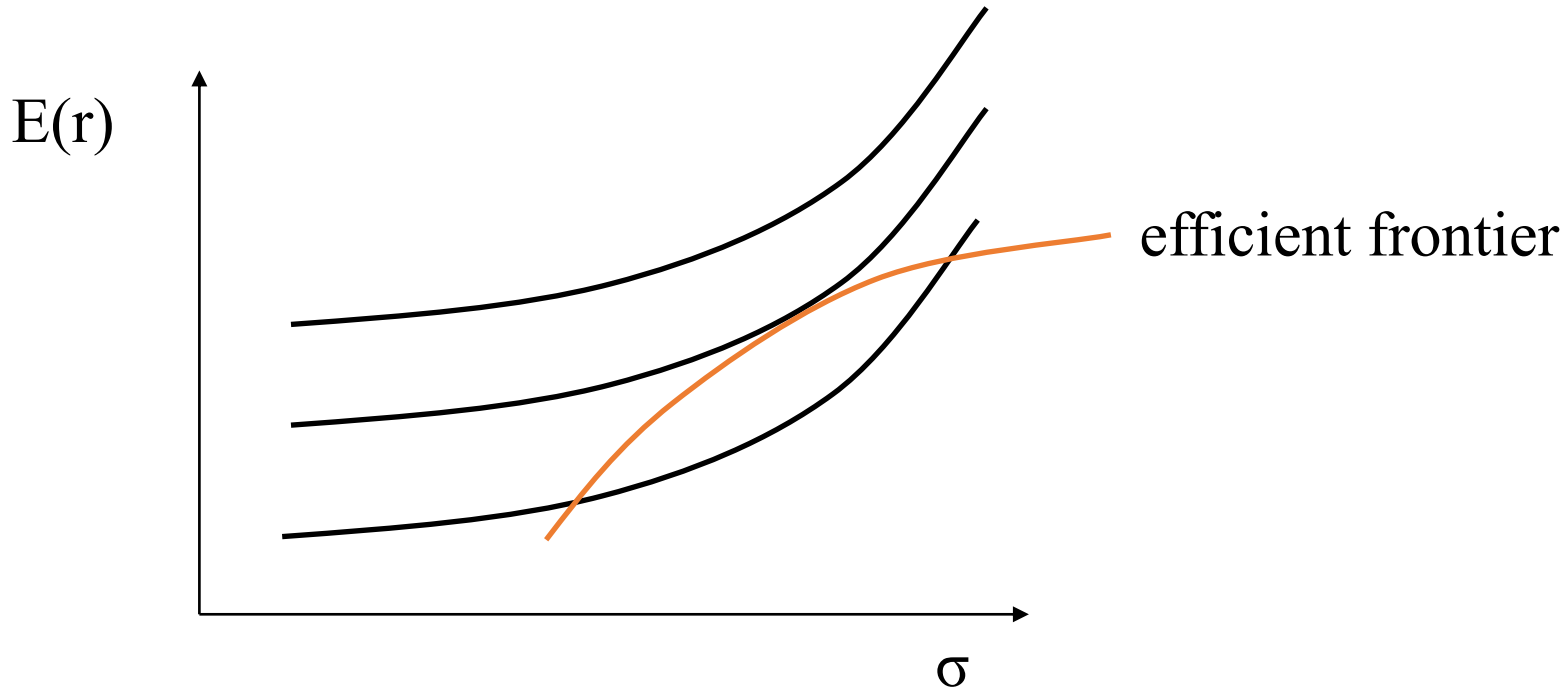
Putting It All Together

An **optimal** portfolio reconciles what is

desirable – described by the *indifference* or utility *curves*

with what is

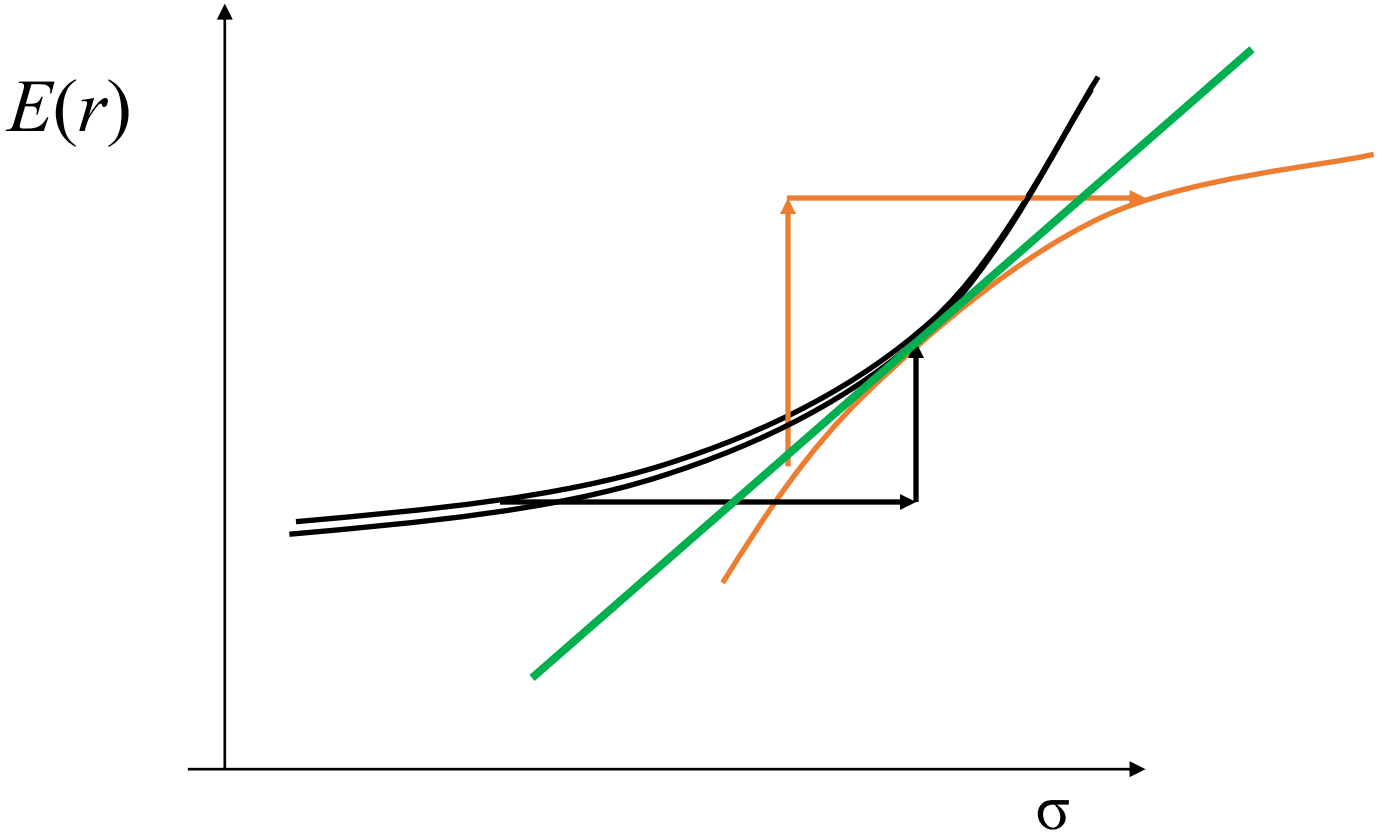
feasible – described by the upper portion of the investment opportunity set, the *efficient frontier*



Optimal Portfolio

The **marginal rate of substitution** is how much investors need to be compensated in terms of higher utility for taking on more risk.

The **marginal rate of transformation** is how much more return the economy can generate when more risk is undertaken.



Conclusion

- Everyone should hold a portfolio on the efficient frontier
- But the location depends on risk aversion

Assignments

- Reading
  - BKM: Chapters 5.4-5.6, 6.3
  - Problems: 5.12-5.14, 5.17, 6.3-6.4, 6.14