Foundations of Finance

Introduction to Bonds

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Fall 2026

Partial Review of the Last Class

- What is a perpetuity worth? And a growing perpetuity?
- Why is an annuity the difference between two perpetuities?
- What is the difference between APR and EAR?
- An investment pays you a fixed amount every year for ten years and then returns your capital. Which two shapes is that made of?

Loose End from Session 3: The Margin Call

- You buy 200 shares at $50, financing half the position with a broker loan
- Initial margin 50%, maintenance margin 30%
- (a) What is your equity today, and how much do you owe?
- (b) How far can the price fall before you get a margin call?
- (c) If the price falls to $30, how much cash must you post?

The Margin Call — Answer

- Position = 200 × \$50 = \$10,000. Loan = \$5,000. Equity = \$5,000
- At any price P: equity = 200P − 5,000, and margin = (200P − 5,000) / 200P
- Margin call when (200P − 5,000) / 200P = 0.30 → 140P = 5,000 → P = \$35.71
- At P = \$30: value \$6,000, equity \$1,000, required 0.30 × 6,000 = \$1,800 → post \$800
- The price falls 28.6% and your equity falls 57%. Leverage is symmetric — that is the point

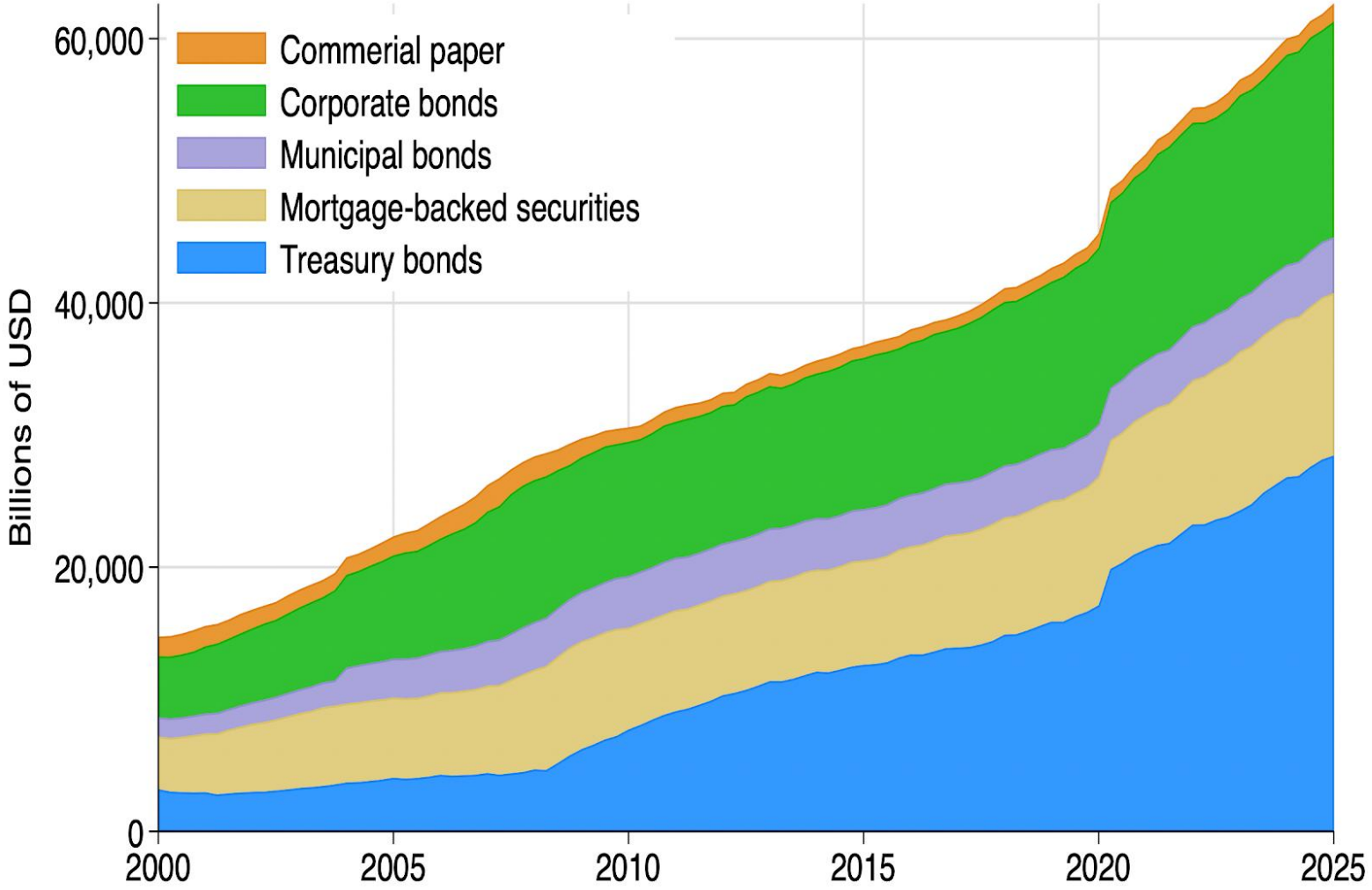
Today

- The bond as an application of present value
- The bond market: the United States, Spain and the euro area
- The main characteristics of a bond
- Constructing the cash flows of a bond
- Zero coupon bonds: pricing, maturity and the pull to par
- Worked example: pricing a bond by decomposition

A Bond Is an Annuity Plus a Zero

- Last class: a perpetuity is worth C / r, and a T-year annuity is worth (C / r)[1 − 1/(1+r)^T]
- A coupon bond pays C every year for T years, and then returns the face value F
- Those are two shapes you already know, added together:
  - **B = (C / r)[1 − 1/(1+r)^T] + F / (1+r)^T**
- An annuity of coupons, plus a zero coupon bond for the principal
- Nothing new today: YTM, price risk and the yield curve are all this one equation, read in a different direction

The U.S. Bond Market



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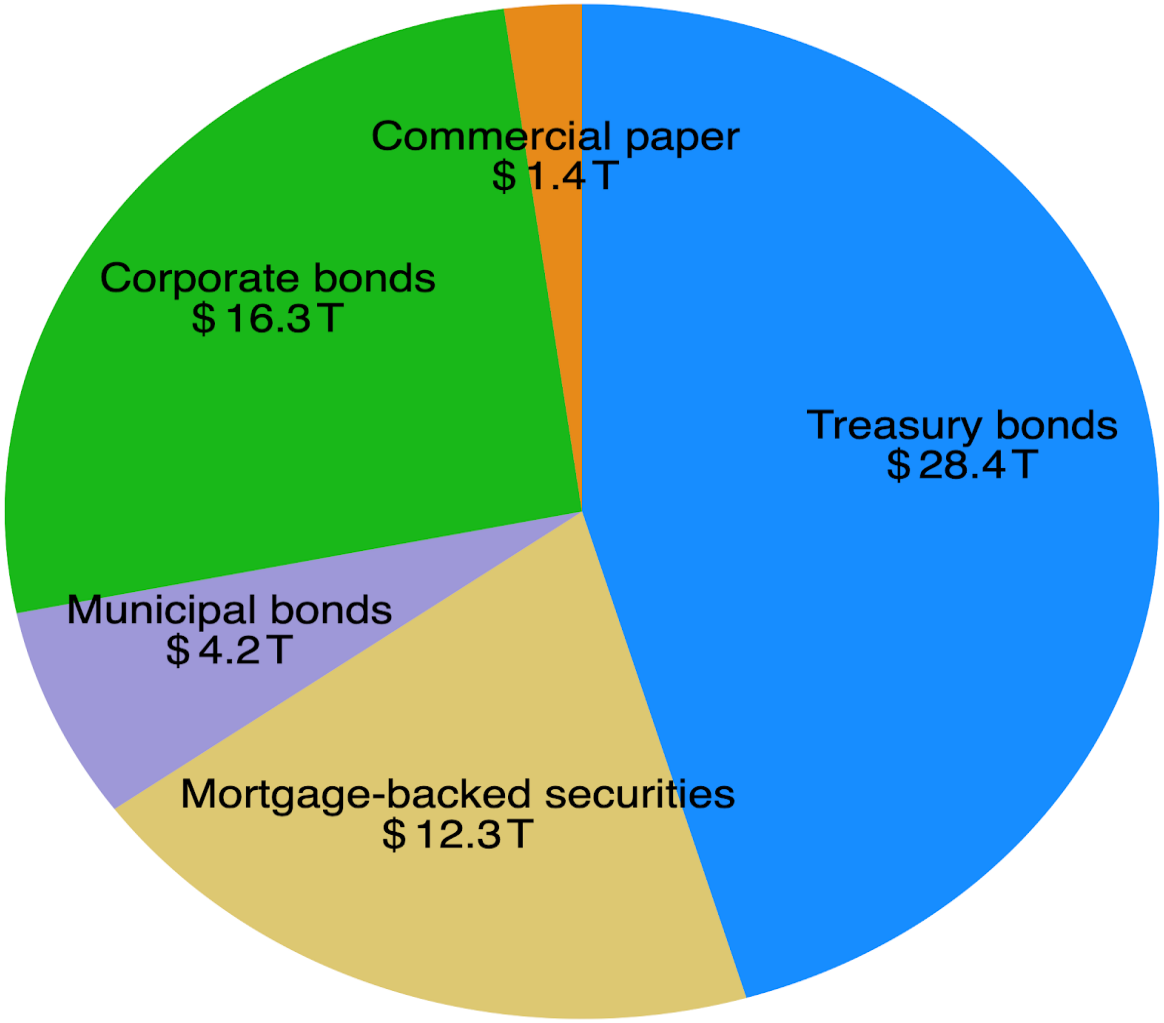


The Spanish and Euro-Area Bond Market

- The Tesoro Público issues in three tranches:
  - Letras del Tesoro — up to 12 months, no coupon, sold at a discount
  - Bonos del Estado — 3 and 5 years, annual coupon
  - Obligaciones del Estado — 10, 15, 30 and 50 years
- A Letra is the Spanish T-bill: exactly the zero coupon bond we price today
- The German Bund is the euro-area benchmark — what Treasuries are in dollars
- One currency, many issuers: euro-area sovereigns are not one credit

La Prima de Riesgo

- Spread = yield on the 10-year Spanish bond − yield on the 10-year German Bund
- Same currency, same maturity: the difference is not payment for time or inflation
- It is payment for the issuer — sovereign credit risk, and liquidity
- Quoted in basis points: 1 bp = 0.01%
- You have met it twice: the sovereign spread in Session 2, Japan and the US in Session 3
- **If the prima de riesgo widens, what has happened to the price of the Spanish bond?**

Who Holds Bonds, and Why

- Insurers and pension funds: long, predictable liabilities, so they buy long assets to match
- Banks: deposits are short and can leave tomorrow; loan and securities books are long
- Central banks: sovereign bonds are the instrument of policy and the collateral of the system
- Households: directly, or far more often through funds and ETFs
- **Note the bank: short liabilities, long assets. Hold that thought — it is Session 7**

Main Characteristics of Bonds

- Issuer:
  - US Government (via the Treasury)
  - States, municipalities and agencies
  - Corporations
  - Foreign governments (sovereign bonds)

- Term: (# of years to maturity)
  - Short (less than 1 year): T bills, CDs, Commercial paper
  - Long (more than 1 year): T-bonds, corporate bonds
  - Consols

Main Characteristics of Bonds

- Seniority and security:
  - Senior, subordinated senior, junior…
  - Secured by property & equipment, other assets of the issuer, income-stream, etc.
  - Sinking fund provisions (sinkers)
- Price vs Par
  - Discount bonds: Price<par
  - Premium bonds: Price>par
- Covenants:
  - Restrictions on additional issues, dividends, and other actions.

Main Characteristics of Bonds

- Coupon rate: total annual interest payments as % of face value
  - Period: annual, semi-annual
  - Fixed or variable
  - Nominal or inflation-protected
- Currency
  - Eurobonds
  - "Yankee" bonds, "Samurai" bonds
- Credit risk
  - Risk that promised payments will not be made
  - Credit default swaps (CDS)

Main Characteristics of Bonds

- Option provisions:
  - Callability: After a certain period, issuer has the right to pay back the loan before it matures.
  - Putability: After a certain period, bondholder has the right to demand payment of the loan before maturity.
  - Convertibility: After a certain period, bondholder had the right to exchange the bond for stocks of the issuer.

Determining the Value and Yield-to-Maturity of a Bond

- The value B of a bond which pays expected coupon payments C_t in each period, has a discount rate r, a maturity T and a face (par) value F is:

$$B = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} + \frac{F_T}{(1+r)^T}$$

- The yield to maturity r (or YTM) of a bond is the discount rate for which the value B of the bond equals the present value of its expected future cash flows
  - The yield to maturity is also referred to as the Internal Rate of Return (IRR)
  - The YTM reflects the rate an investor earns per year if the bond is held to maturity

Constructing Cash Flows of a Bond

- From the perspective of valuation, first thing we need to figure out are the promised cashflows

- The promised cashflows are characterized by:
  - Principal amount (or face value): The balloon payment to be made at maturity.
  - Coupon payment (or coupon rate):
    - Coupon can be fixed or variable
    - Coupon can also be zero, as we have seen
    - Coupon can be indexed: to inflation, to other outcomes, say pandemics
  - Coupon payment frequency

Single Payment Security

- Zero coupon bond
  - A bond that pays no interest/coupon, only the face value at maturity.

- Where do zero coupon bonds come from?
  - Issued in primary markets (Treasury bills)
  - Stripping of coupon bonds (example of "financial engineering")

- Pricing: Price = PV = $F/(1+r)^t$
  - Lower interest rates → Higher bond prices.
  - Note the "pull to par" of prices as we approach maturity.

Pricing Zero Coupon Bonds

- Let us compare two zero coupon bonds. Both have a face value of $1000 and interest rate of 5% and 2% respectively

Maturity	r=8%	r = 5%	r = 2%
T=10			
T=5			
T=2			
T=1			

- Lower interest rates → higher bond prices
- Higher maturity → lower bond prices

Pricing Zero Coupon Bonds — Answer

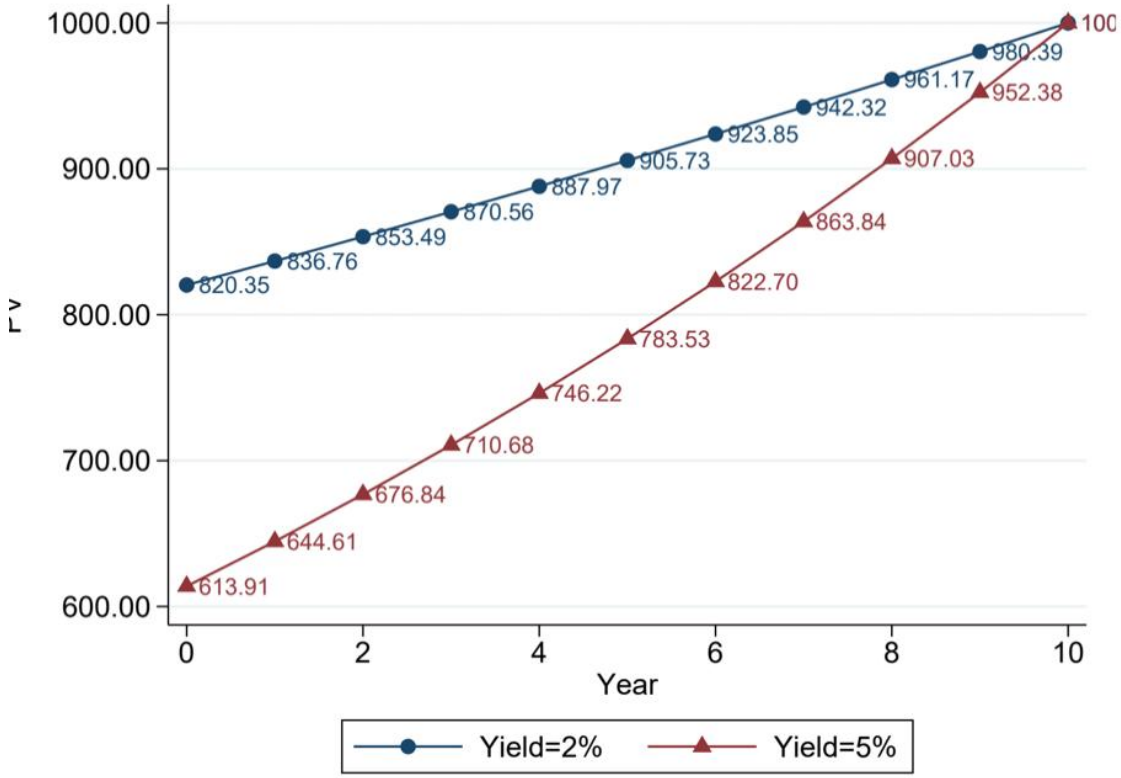
- Let us compare two zero coupon bonds. Both have a face value of $1000 and interest rate of 5% and 2% respectively

Maturity	r=8%	r = 5%	r = 2%
T=10	463.19	613.91	820.35
T=5	680.58	783.53	905.73
T=2	857.34	907.03	961.17
T=1	925.93	952.38	980.39

- Lower interest rates → higher bond prices
- Higher maturity → lower bond prices

Single Payment Security-Zero Coupon Bond

- Prices of a zero coupon bond with face value $1000 maturing in year 10. Calculations for yields of 2% and 5%.



Worked Example: Pricing by Decomposition

- A 3-year bond, face value $1,000, annual coupon of 6%, required return r = 5%
- (a) What is the price of a 3-year zero with the same face value?
- (b) What are the three $60 coupons worth today?
- (c) What is the price of the coupon bond?
- (d) Discount, par or premium — and why?
- (e) Now set r = 6%. Predict the price before you compute it

Pricing by Decomposition — Answer

- (a) The zero: $1{,}000 / 1.05^3 = \$863.84$
- (b) The coupons are an annuity: $(60 / 0.05)[1 - 1/1.05^3] = \$163.39$
- (c) $B = 863.84 + 163.39 = \$1{,}027.23$
- (d) Premium: the coupon rate (6%) is above the required return (5%), so the bond pays more than the market is asking for
- (e) At $r = 6\%$ the price is exactly \$1,000 — coupon rate = required return = par
- **The bond was never a new formula: an annuity plus a zero**

Summary

- A bond is a contract: face value, coupon, maturity, issuer — plus seniority, options, currency
- Its price is the PV of its promised cash flows: an annuity of coupons plus a zero
- The zero is the atom: P = F / (1+r)^T. Everything else is built out of it
- Longer maturity and higher yields push the price down; prices pull to par at maturity
- The same arithmetic runs through the Tesoro, the Bund and the prima de riesgo
- **Next class: Bond Valuation (BMAE 3.2–3.3) — turn the equation around and solve for the yield**