

Foundations of Finance

Bond Valuation

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Fall 2026

Partial Review of the Last Class

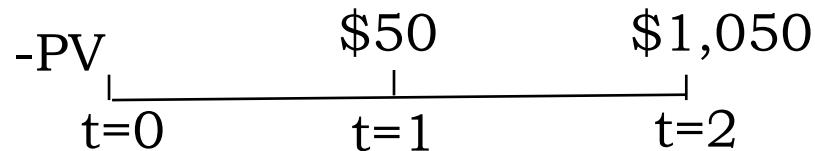
- ▶ What is a 3-year zero with face value \$1,000 worth at a yield of 5%?
- ▶ Which four things define the cash flows of a bond?
- ▶ Why is a coupon bond an annuity plus a zero?
- ▶ Interest rates rise tomorrow. What happens to the price of the bond you already own — and why?

Today

- ▶ Yield to maturity: the bond's internal rate of return
- ▶ Discount, par and premium bonds
- ▶ Semi-annual coupons — the convention almost every real bond uses
- ▶ Price and interest rates: the iron law of fixed income
- ▶ Holding period returns
- ▶ Yield versus return: why the YTM is usually not what you earn

Valuing a Coupon Bond

- ▶ What is the PV of a \$1,000 par, 2-year, 5% coupon bond (annual payments)



- ▶ Compute the present value that will earn 5% yield?

$$PV = \frac{50}{1.05} + \frac{1050}{(1.05)^2} = 47.62 + 952.38 = 1,000$$

- ▶ Could you have guessed this outcome?
- ▶ Compute the present value that will earn 6% yield?

Yield to Maturity (YTM) on Annual Pay Coupon Bonds

- ▶ For an annual-pay coupon bond, the YTM is calculated as the Internal Rate of Return
- ▶ It is the average rate of return
- ▶ Hence, YTM is the rate (y) that solves:

$$P_0 = \sum_{t=1}^T \frac{C_t}{(1 + YTM)^t} + \frac{F_T}{(1 + YTM)^T}$$

- ▶ Note that the coupon is paid in the last period too!

Example

- ▶ 3-year annual pay coupon bond, with coupon rate of 8%, face value of \$1,000
- ▶ Draw cash-flow line
- ▶ In the bond sells at $P=\$1,000$, what is YTM?
- ▶ YTM solves the pricing equation:

$$1000 = \frac{80}{(1+YTM)^1} + \frac{80}{(1+YTM)^2} + \frac{1080}{(1+YTM)^3}$$

Example — Answer

- ▶ 3-year annual pay coupon bond, with coupon rate of 8%, face value of \$1,000
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- ▶ Answer: YTM=coupon rate=8% (par bond)
- ▶ Bond price equals face value—bond is trading *at par*

Discount, Par and Premium Bonds

- [illegible]

Discount, Par and Premium Bonds — Answer

- ▶ If the bond sells at \$1,000, what is the YTM?
- ▶ If the bond sells at \$900, what is the YTM?
- ▶ If the bond sells at \$1,100, what is the YTM?

	Price	Coupon	Current Yield	YTM
Premium Bond	1,100	8%	7.27%	4.35%
Par Bond	1,000	8%	8%	8%
Discount Bond	900	8%	8.88%	12.17%

- ▶ **If the YTM is higher than the coupon rate, then what must be true about the price?**

Semi-Annual Coupons

- ▶ Almost every real bond pays twice a year: US Treasuries, Obligaciones del Estado, most corporates
- ▶ The convention is mechanical: halve the coupon, halve the yield, double the periods
 - ▶ $B = \sum (C/2) / (1 + y/2)^t + F / (1 + y/2)^{(2T)}$
- ▶ So a quoted bond yield is an APR, not an EAR — the distinction from Session 4
 - ▶ $EAR = (1 + y/2)^2 - 1$
- ▶ The most common error: halving the coupon and forgetting to halve the yield

Semi-Annual Coupons — Example

- ▶ A 3-year bond, face value \$1,000, 8% coupon paid semi-annually, quoted yield 6%
- ▶ Per period: coupon \$40, yield 3%, six periods
- ▶ $B = 40 \times (1/0.03)[1 - 1/1.03^6] + 1,000 / 1.03^6 = 216.69 + 837.48 = \$1,054.17$
- ▶ The effective annual yield is $1.03^2 - 1 = 6.09\%$, not 6%
- ▶ The same bond quoted 6% annual-pay would be worth \$1,053.46 — close, but not the same bond

Bond Price Changes as a Function of Coupon Rate

- ▶ Value of 3-year annual pay coupon bonds with face value of \$1,000 and different coupons and YTM

	4%YTM	8%YTM	12%YTM
4% coupon	1,000.00	896.91	807.85
% change	11.5%	0.0%	-9.9%
8% coupon	1,111.00	1,000.00	903.93
% change	11.1%	0.0%	-9.6%
12% coupon	1,222.01	1,103.08	1,000.00
% change	10.78%	0.0%	-9.34%

Summary: The Impact of Interest Rates on Bond Values

- ▶ **Bond values and interest rates are inversely related**
 - ▶ When the interest rates goes up, the value of a bond goes down, and vice versa. This is also called the “Iron Law of Fixed Income”
- ▶ **Even in the absence of default risk, every bond has interest rate risk**
 - ▶ For zero coupon bonds, this interest rate risk is price risk
 - ▶ Coupon bonds have both price risk and reinvestment risk
- ▶ **The interest rate risk of a bond can be measured by how strongly the value of the bond changes with the interest rate**
 - ▶ The price sensitivity of a bond to the interest rate depends on key bond features, such as the remaining time to maturity on the bond and the coupon rate, as well as the level of the interest rate in the market

Holding Period Returns

- ▶ The holding period return (HPR) is the rate (return) that your initial investment earned in order to generate the final value of the investment
- ▶ Suppose that:
 - ▶ You invest V_0 at time 0
 - ▶ You reinvest all intermediate cash flows (dividends, coupons, etc.)
 - ▶ At date T , the investment (including reinvested cash flows) is worth V_T
- ▶ Then the $HPR = \frac{V_T}{V_0} - 1$

Annualized HPR

- ▶ To compare returns across time periods, need a measure of return per year (or other unit of time).
- ▶ The annual holding period return (ann. HPR) is the corresponding rate per year (annual return)
- ▶ Let T measure the length of the holding period in years, and V_0 the initial value of an investment, and V_T the ending value.
- ▶ The annual holding period return (ann. HPR) :

$$\text{annHPR} = \left(\frac{V_T}{V_0} \right)^{1/T} - 1$$
$$\text{annHPR} = (1 + \text{HPR})^{1/T} - 1$$

HPR for Zero Coupon Bond

- ▶ You buy a 3-year zero-coupon bond with a face value of \$100 and 5% yield to maturity
- ▶ What is the bond's current price?
- ▶ If you hold the bond to maturity, what is your holding period return?
- ▶ What is your annualized holding period return?

HPR for a Zero Coupon Bond — Answer

- ▶ Price = $100 / 1.05^3 = \$86.38$
- ▶ Held to maturity: HPR = $100 / 86.38 - 1 = 15.76\%$ over the three years
- ▶ Annualised: $(1.1576)^{(1/3)} - 1 = 5.00\%$ — exactly the yield to maturity
- ▶ A zero has no coupons, so there is nothing to reinvest and nothing to go wrong
- ▶ **Hold a zero to maturity and the YTM is what you earn. It is the only case in which that is guaranteed**

Yield vs Return on Bond

- ▶ If you invest in a bond, will you always earn its yield to maturity?
- ▶ Your annualized holding period return will equal the bond's yield to maturity only if
 - ▶ You hold the bond to maturity
 - ▶ You re-invest all coupons at that yield to maturity
 - ▶ There's no default
- ▶ If you sell a bond early or re-invest coupons at a different rate, you will earn a different return ($ann. HPR \neq YTM$)
- ▶ Bonds are risky even if their cash flows are fixed

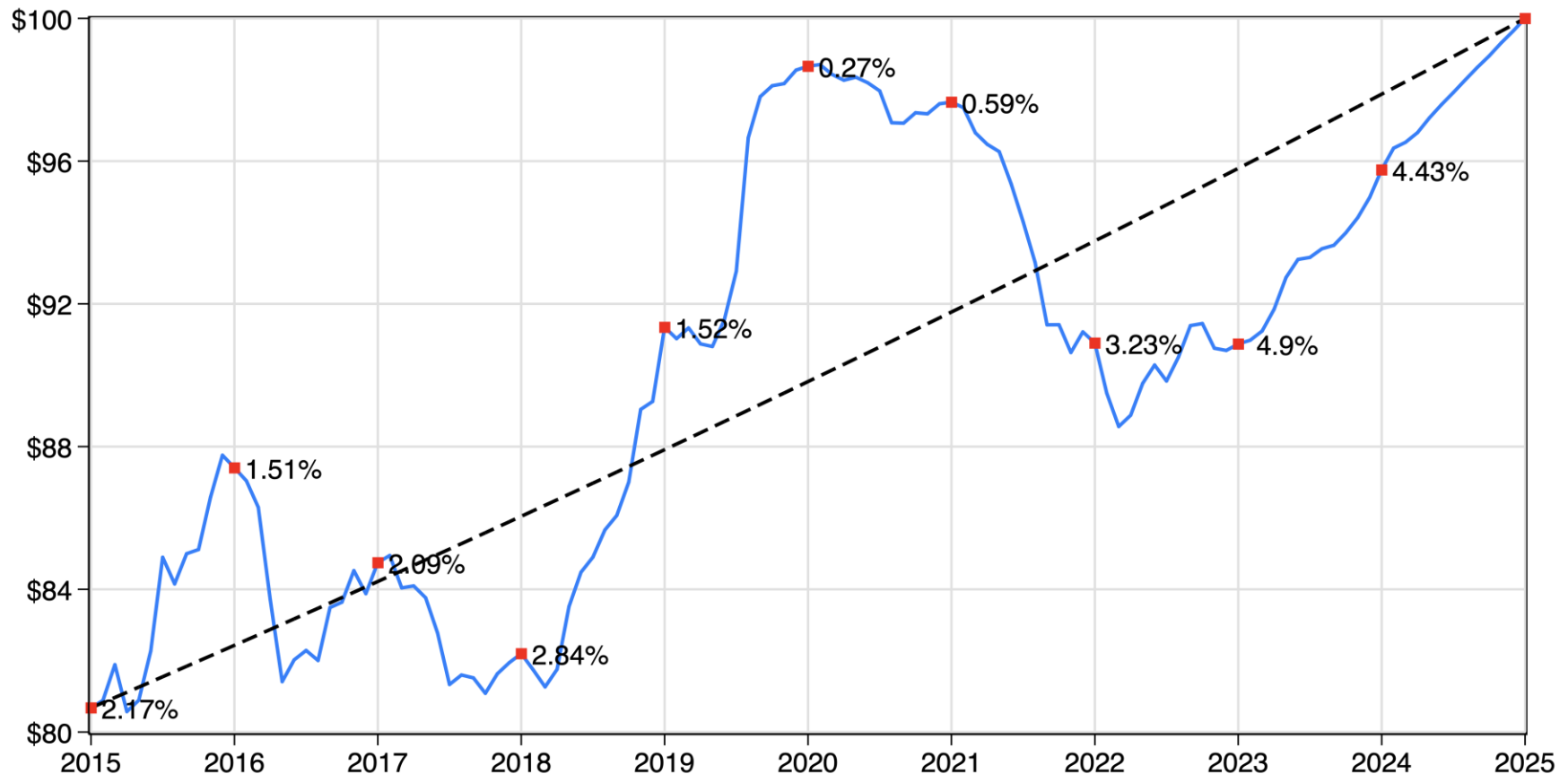
HPR on Zero-coupon Bond

- ▶ Suppose you buy a 3-year zero-coupon bond with \$1000 face value. Current YTM is 5%.
 - ▶ What is the bond's current price?
- ▶ Next year, at $t = 1$ there are two years remaining to maturity. The new YTM is 7%.
 - ▶ What is the price now?
 - ▶ What is the realized holding period return over the one year period?
- ▶ Suppose instead the YTM stayed at 5%.
 - ▶ What would be the price that year?
 - ▶ What would be the realized return over the one year period?

HPR on a Zero Coupon Bond — Answer

- ▶ Today: $P_0 = 1,000 / 1.05^3 = \$863.84$
- ▶ Yields rise to 7%: $P_1 = 1,000 / 1.07^2 = \$873.44 \rightarrow \text{realised return} = 873.44 / 863.84 - 1 = 1.11\%$
- ▶ Yields unchanged at 5%: $P_1 = 1,000 / 1.05^2 = \$907.03 \rightarrow \text{realised return} = 5.00\%$
- ▶ You bought a 5% yield and earned 1.11%. The yield was never a promise about any one year
- ▶ **Sell before maturity and your return is the market's opinion on the day you sell**

Performance of a 10-year ZCB Issued 10 Years Ago



HPR on an Annual-pay Coupon Bond

- ▶ A 4-year coupon bond has face value of \$1000, coupons of \$80, and a YTM of 8%
 - ▶ What is the bond's current price?
 - ▶ What is the bond's future value *if coupons can be reinvested each year at YTM = 8%*?
 - ▶ What is the bond's annual return?
- ▶ What if next year, the interest rate falls to 4%?
 - ▶ What is the bond's future value?
 - ▶ What is the bond's annual return?

HPR on an Annual-Pay Coupon Bond — Answer

- ▶ Price today = \$1,000: coupon rate equals YTM, so the bond is at par
- ▶ Coupons reinvested at 8%: $FV = 80 \times [(1.08^4 - 1)/0.08] + 1,000 = \$1,360.49$
- ▶ Annual return = $(1,360.49 / 1,000)^{(1/4)} - 1 = 8.00\% = \text{the YTM}$
- ▶ If rates fall to 4%, coupons compound at 4%: $FV = 339.72 + 1,000 = \$1,339.72$
- ▶ Annual return = 7.59%, not 8% — and nothing defaulted. This is reinvestment risk
- ▶ **A zero has price risk only. A coupon bond has price risk and reinvestment risk**

Summary

- ▶ The YTM is the IRR of the bond's promised cash flows — an average, not a promise
- ▶ Coupon above YTM → premium; equal → par; below → discount. Arithmetic, not convention
- ▶ Quoted yields are APRs: halve the yield, double the periods for a semi-annual bond
- ▶ The iron law: rates up, prices down — more so the longer the maturity
- ▶ You earn the YTM only if you hold to maturity, reinvest at that yield, and nothing defaults
- ▶ **Next class: Interest Rate Risk and the Yield Curve (BMAE 3.4–3.5)**