Foundations of Finance

Risk and Return

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Fall 2026

Where We Are: The Second Half

- First half: value = PV of expected cash flows
  - Bonds, stocks, projects: the discount rate was given
- Second half: where does the discount rate come from?
  - Today: measuring risk and return
  - Next: portfolios and diversification
  - Then: which risk is rewarded (CAPM), cost of capital
- Finally: capital structure, payout policy and options

Introduction

- We value investments by discounting their cash flows at the required return of investors
  - The required return is the rate investors expect to earn on other investments with similar risk
  - It ensures that investors are compensated for their opportunity cost
- Factors affecting required returns
  - Time Value of Money: captured by the risk-free interest rate R_f
  - Risk: investors are risk averse, so riskier investments have higher required returns
- We need a framework to determine the risk of an investment and translate it into a required return

Revisiting Holding Period Return (HPR)

- To calculate the return on any investment, you need
  1. The initial value/cost of the investment, V_0 $(= PV)$
  2. The final value of the investment, V_T $(= FV)$, including all intermediate cash flows (coupons, dividends) and their re-investment gains
- The annualized holding period return is the equivalent per period rate of return: $V_0(1 + \text{ann.HPR})^T = V_T$

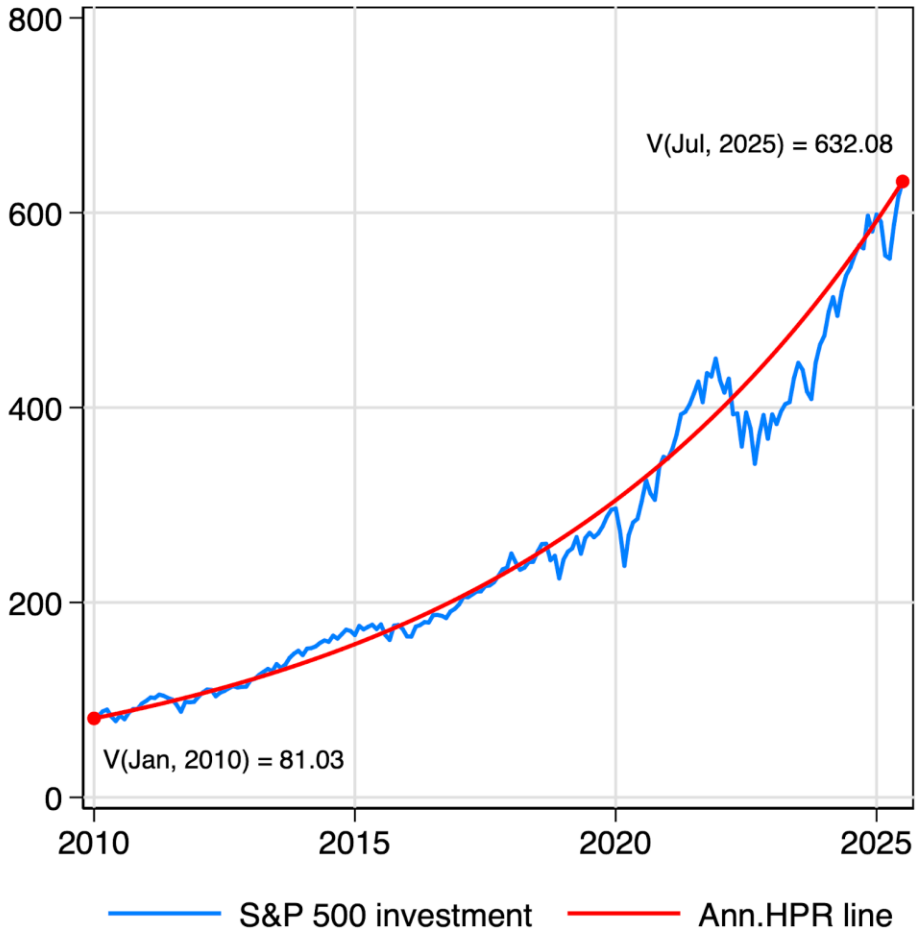
$$\text{ann.HPR} = \left(\frac{V(T)}{V(0)}\right)^{1/T} - 1$$

- The holding period return (not annualized) is $\text{HPR} = \frac{V_T}{V_0} - 1$

S&P 500 Annualized HPR since 2010

- If you bought S&P 500 in Jan 2010 and held it while reinvesting dividends until July 2025 (186 months), your annualized return was

$$ann.HPR = \left(\frac{632.08}{81.03}\right)^{\frac{1}{186/12}} - 1 = 14.17\%$$



One-period HPR on a Security

- Consider a security (stock or bond) that pays income (dividend or coupon) each period:

$$HPR = \frac{Income + Ending\ Price}{Beginning\ Price} - 1$$

- The return is the sum of two components:
  - Income: the current dividend or coupon
  - Capital gain: the change in the price (due to changes in the value of future income)
- The income component is called the dividend yield (D/P) for stocks and the current yield (C/P) for bonds

HPR Stock Example

- You bought Apple shares for $144.02 two years ago and sold them for $173.58. Assume that the only dividend of $2.92 is paid at the end of the second year.

- What are the HPR and ann. HPR?

Multi-period Realized Return

- An emerging markets fund has a 100% return in year 1 and a −50% return in year 2
  - We forecast next year's return to be $(+100\% - 50\%)/2$ (arithmetic average)
- \$100 invested grows to \$100, so annualized HPR is

$$(100/100)^{1/2} - 1 = 0\%$$

- Shorter, use geometric average of gross returns:

$$(2x0.5)^{1/2} - 1 = 0\%$$

- Without reinvesting gains/ losses the investor would have gained \$100 in year 1 and lost \$50 in year 2, or \$50 in total over two years. This 25% is the arithmetic average. The geometric average is for an initial sum left to grow untouched by human hands.

Arithmetic vs. Geometric Returns

- The arithmetic average return does not account for re-investing
  - Useful for forecasting next period's return
- The geometric average return accounts for re-investing
  - Equals the $ann.HPR$ of an investor who re-invests the income (dividend, coupon) in the same asset
- Volatility drag: when returns are volatile, the geometric average is lower than the arithmetic average
  - Due to asymmetry of positive vs negative returns: it takes a larger percentage gain to offset a given percentage loss

Multi-Period HPR Example

- A hedge fund reported following returns for the first three months.
- Calculate HPR and annHPR

Jan	1.90%
Feb	2.73%
Mar	2.04%

Ex. Apple, Inc. Quarterly Returns

2024	Beginning Price	Ending Price	Dividend	HPR
Q1	$192.53	$171.48	$0.24	-10.8%
Q2	$171.48	$210.62	$0.25	23.0%
Q3	$210.62	$233.00	$0.25	10.7%
Q4	$233.00	$250.42	$0.25	7.6%

$$= \frac{0.25 + 233}{210.62} - 1$$

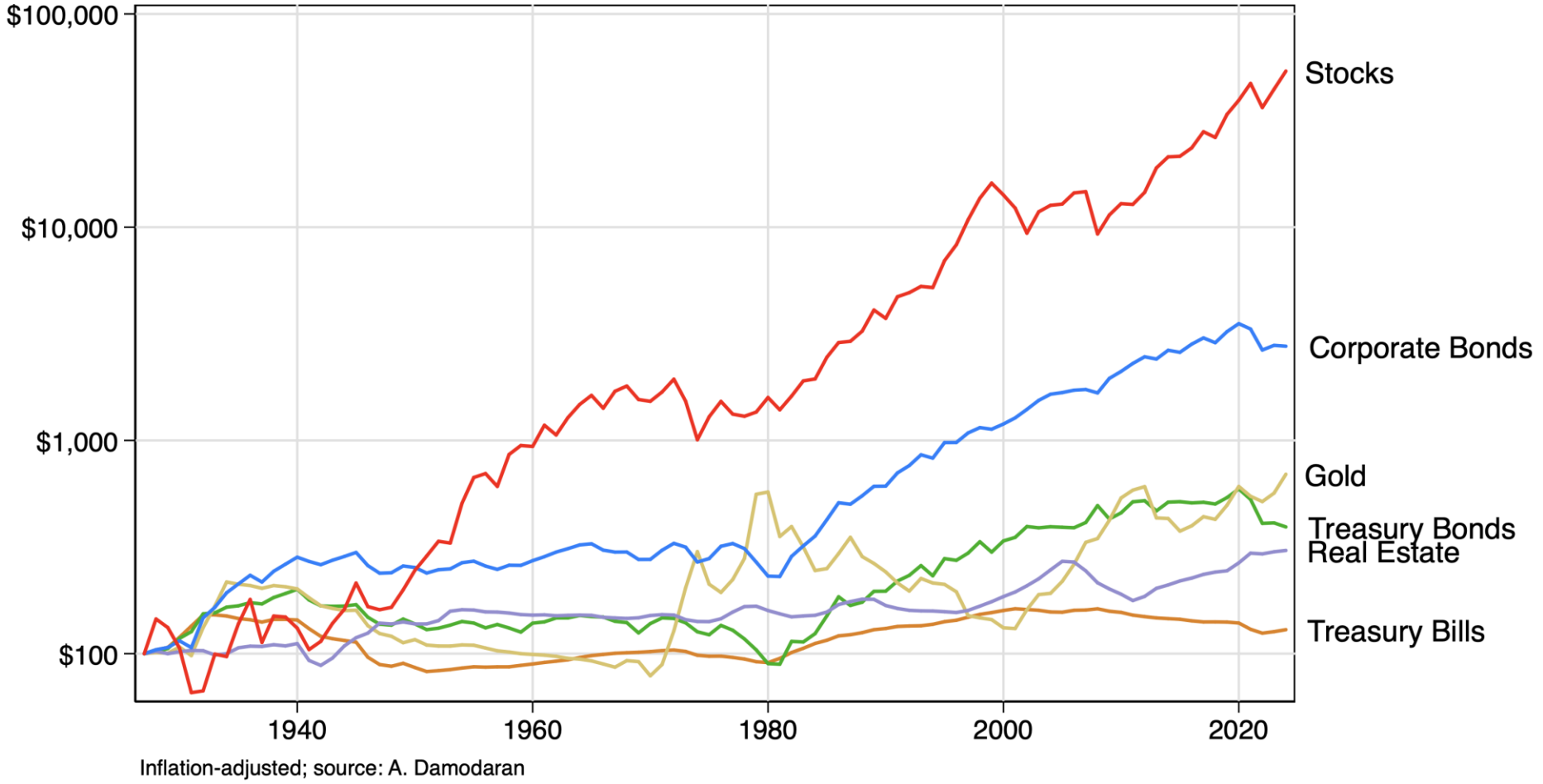
- Apple's arithmetic average return was:

$$= \frac{1}{4}(-0.108 + 0.230 + 0.107 + 0.076) = 7.6\%$$

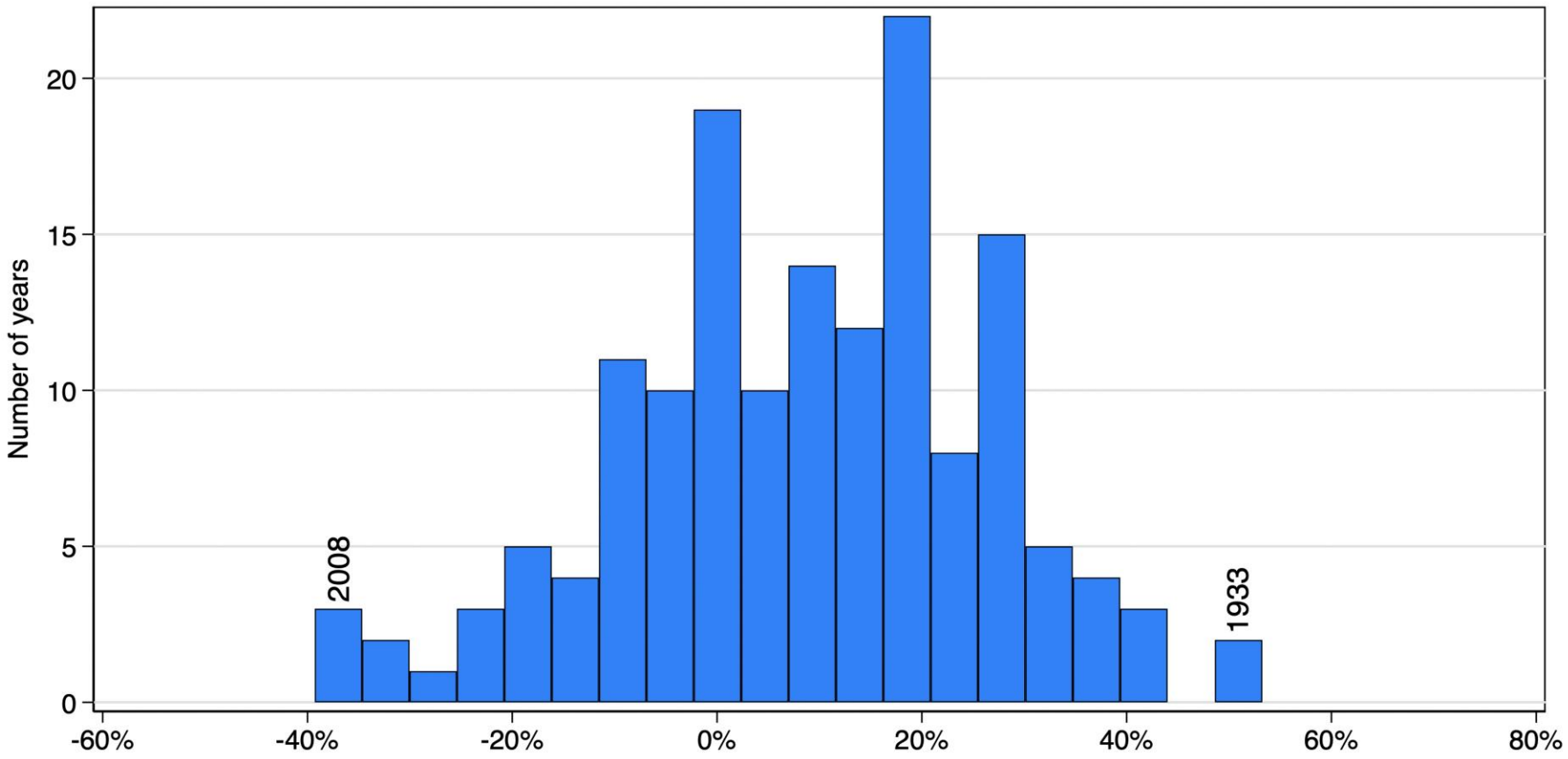
- Apple's geometric average return was:

$$= [(1 - 0.108)(1 + 0.230)(1 + 0.107)(1 + 0.076)]^{1/4} - 1$$
$$= 6.9\%$$

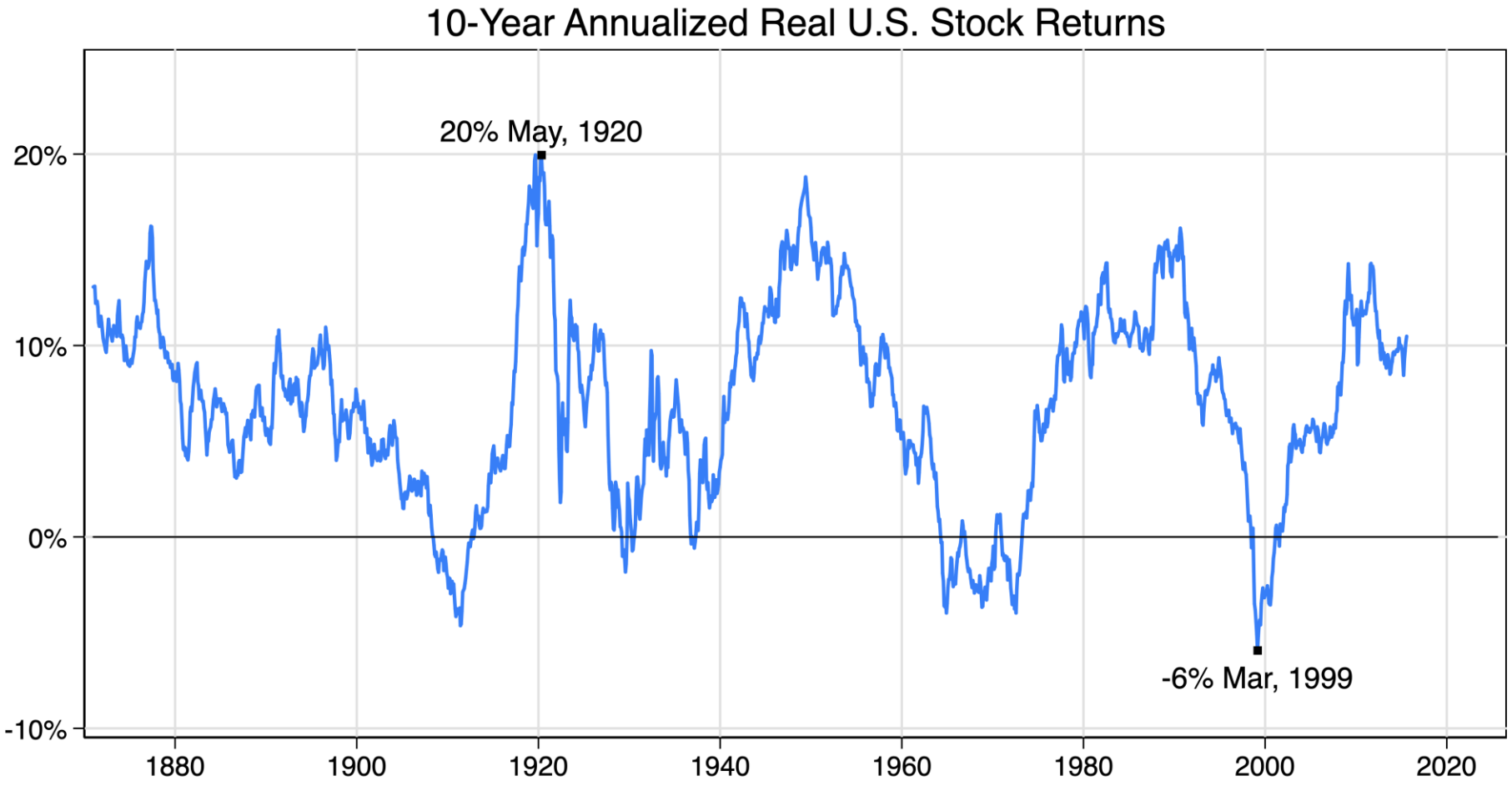
Historic Performance of Different Asset Classes



Distribution of US Stock Returns since 1870



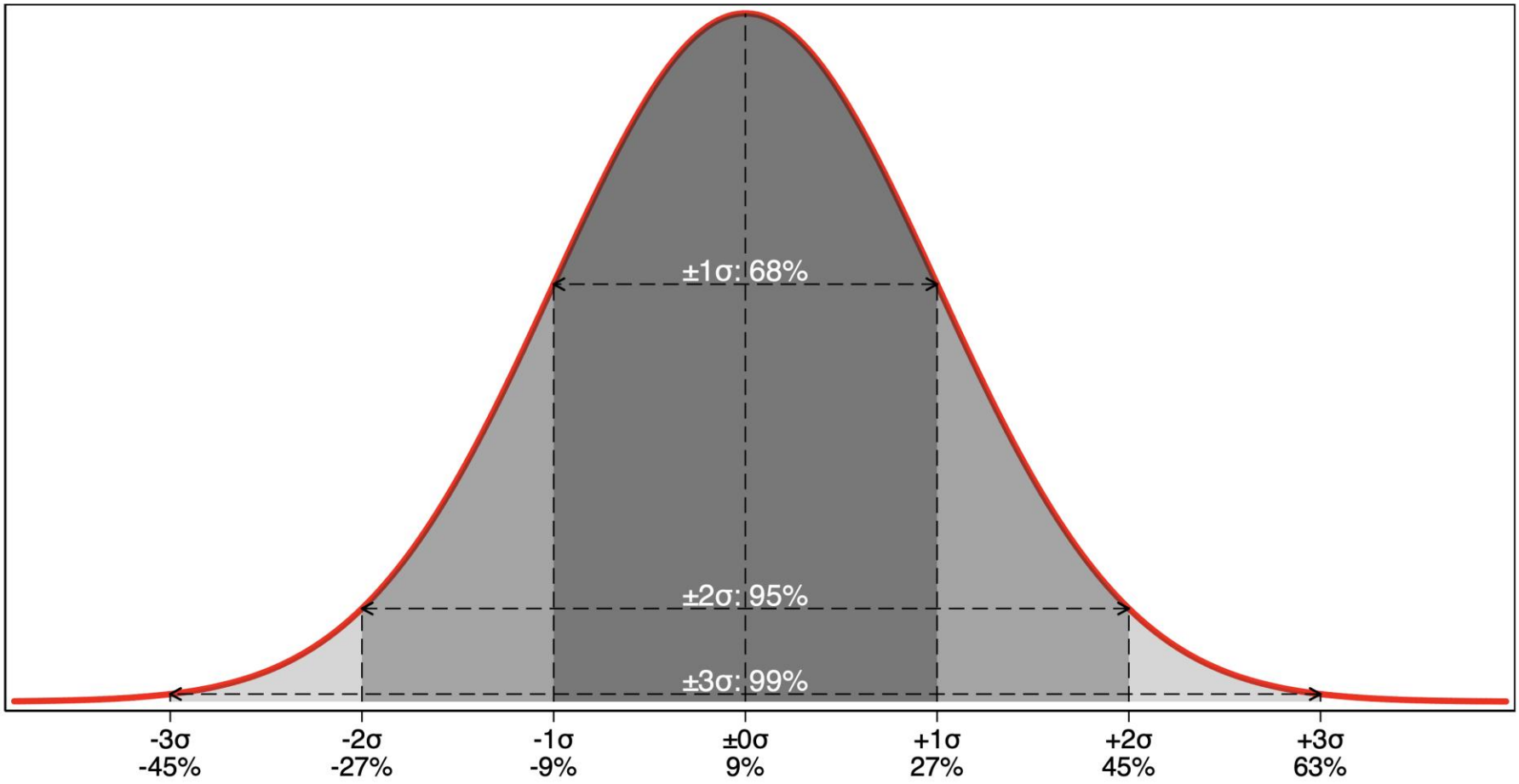
Are Stocks Safe in the Long Run?



How to Measure Risk?

- A risky investment is one whose returns are uncertain
- To quantify the risk of an investment, we need to analyze the probability distribution of its returns
  - Can be done on a forward-looking basis using scenario analysis
  - Or a backward-looking basis using data on past returns (time-series analysis)
- When returns follow a normal distribution, they are fully described by two parameters:
  - Expected (mean) return
  - Standard deviation of returns
  - With two investments, we also need their covariance or correlation
- Mean-variance analysis and modern portfolio theory

Applying a Normal Distribution to US Stock Returns



Expected (Mean) Return

- Suppose the possible future states of the world (scenarios or outcomes) are $s = 1, 2, \ldots, S$, where
  - The probability of state s is $\mathrm{p}(s)$
  - The return of an investment in state s is $\mathrm{R}(s)$
- The *expected (or mean) return* of the investment is its probability-weighted average return:

$$E[R] = p(1) \times R(1) + p(2) \times R(2) + \cdots$$

$$= \sum_{s=1}^{S} p(s) \times R(s)$$

  - Equals the average return if repeated infinitely many times

Ex. Expected Return

State of economy s	Probability $p(s)$	Walmart return R_{WMT}	Target return R_{TGT}
Recession	10%	0%	-10%
Normal	50%	10%	10%
Boom	40%	20%	30%

$$E[R_{WMT}] = 0.1 \times 0 + 0.5 \times 0.1 + 0.4 \times 0.2 = 13\%$$

$$E[R_{TGT}] = 0.1 \times (-0.1) + 0.5 \times 0.1 + 0.4 \times 0.3 = 16\%$$

Standard Deviation of Returns

- The *variance* of returns, σ^2, is the expected squared deviation from the mean:

Squared deviation from the mean

$$Var[R] = \sigma^2$$
$$= p(1) \times (R(1) - E[R])^2 + p(2) \times (R(2) - E[R])^2 + \cdots$$
$$= \sum_{s=1}^{S} p(s) \times (R(s) - E[R])^2$$

- The *standard deviation* or *volatility*, σ, is the square root of the variance:

$$StDev[R] = \sqrt{Var[R]} = \sigma$$

- Captures the variability of returns around their mean

Ex. Standard Deviation

State of economy s	Probability $p(s)$	Walmart return R_{WMT}	Target return R_{TGT}
Recession	10%	0%	-10%
Normal	50%	10%	10%
Boom	40%	20%	30%

$$\sigma^2_{WMT} = 0.1 \times (0 - 0.13)^2 + 0.5 \times (0.1 - 0.13)^2 + 0.4 \times (0.2 - 0.13)^2$$
$$= 0.0041$$
$$\sigma_{WMT} = \sqrt{0.0041} = 6.4\%$$

$E[R_{WMT}]$

Covariance of Returns

- The *covariance*, σ_{ij}, of the returns of assets i and j is the expected product of their deviations from their means

$$Cov[R_i, R_j] = \sigma_{ij}$$

$$= \sum_{s=1}^{S} p(s) \times (R_i(s) - E[R_i]) \times (R_j(s) - E[R_j])$$

Product of deviations

- Captures their tendency to fluctuate together
  - Positive covariance: the two assets tend to deliver higher-than-expected returns at the same time. E.g. stocks in the same sector
  - Negative covariance: when one asset's return is higher than expected, the other tends to be lower than expected (and vice versa). E.g. stocks and gold (flight to quality)

Correlation

- The *correlation*, ρ_{ij}, is the covariance divided by the product of the standard deviations:

$$Corr[R_i, R_j] = \rho_{ij} = \frac{Cov[R_i, R_j]}{\sigma_i \sigma_j}$$

- It measures the tightness of the relationship between the two assets
  - Has the same sign as the covariance
- The correlation is always between -1 and $+1$:

$$-1 \leq \rho_{ij} \leq +1$$

Ex: Covariance and Correlation

State of economy s	Probability $p(s)$	Walmart return R_{WMT}	Target return R_{TGT}
Recession	10%	0%	-10%
Normal	50%	10%	10%
Boom	40%	20%	30%

$E[R_{WMT}]$

$E[R_{TGT}]$

$$Cov[R_{WMT}, R_{TGT}] = \sigma_{WMT,TGT} = 0.1 \times (0 - 0.13) \times (-0.1 - 0.16) + 0.5 \times (0.1 - 0.13) \times (0.1 - 0.16) + 0.4 \times (0.2 - 0.13) \times (0.3 - 0.16) = 0.0082$$

$$Corr[R_{WMT}, R_{TGT}] = \rho_{WMT,TGT} = \frac{0.0082}{0.064 \times 0.1281} = 1$$

σ_{WMT}

σ_{TGT}

Time Series Analysis

- Instead of projecting forward-looking scenarios, we can look at historical returns. Suppose we have data on past realized returns R_t for period $t = 1, 2, \ldots T$.
- The sample mean (average) return is

$$\widehat{E[R]} = \frac{1}{T}\sum_{t=1}^{T} R_t$$

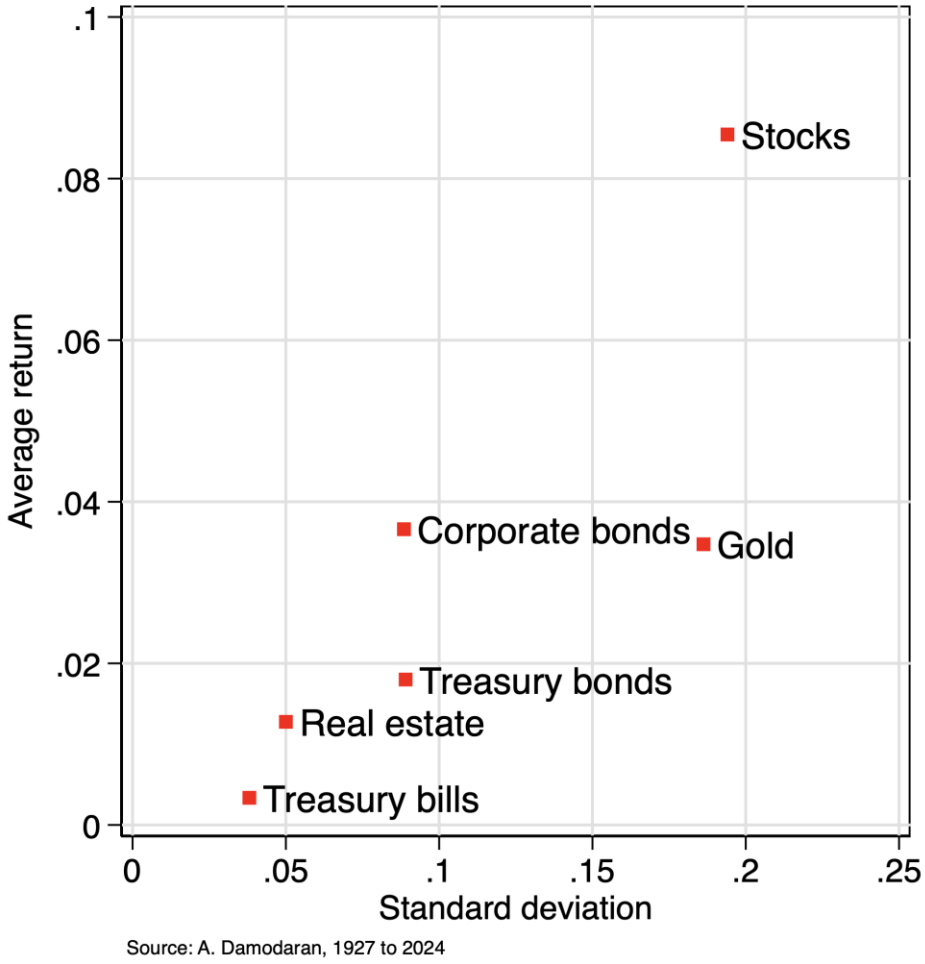
- The sample variance and standard deviation are

$$\widehat{Var[R]} = \frac{1}{T-1}\sum_{t=1}^{T}\left(R_t - \widehat{E[R]}\right)^2 \; and \; \hat{\sigma} = \sqrt{\widehat{Var[R]}}$$

- This approach works if (1) we have enough data and (2) the data is representative of the full distribution (unbiased sample)

The Risk-Return Tradeoff

- Investors are risk averse
- They demand higher expected returns from riskier assets
- Risk premium: the difference between an asset's expected return and the T-Bill rate
- Investors therefore face a risk-return tradeoff



Source: A. Damodaran, 1927 to 2024