Foundations of Finance

The Capital Asset Pricing Model (CAPM)

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Today's Agenda

- Diversification with many assets
- Systematic and idiosyncratic risk
- The CAPM: assumptions and competitive equilibrium
- The market portfolio and the Capital Market Line
- Beta and the Security Market Line
- Practice questions

Partial Review of the Last Class

- What is a Capital Allocation Line? What is its slope?
- What is the two-fund separation theorem?
- What is minimum variance efficient portfolio?

Portfolio Variance and SD

- Now let's revisit our portfolio formulas in the case of many assets.
- With two securities, the portfolio variance is:

$$\sigma_p^2 = \omega_1^2\sigma_1^2 + \omega_2^2\sigma_2^2 + 2\omega_1\omega_2\rho_{12}\sigma_1\sigma_2$$

- With N securities, the portfolio variance is:

$$\sigma_p^2 = \sum_{i=1}^{N}\omega_i^2\sigma_i^2 + 2\sum_{i=1}^{N}\sum_{j>i}\omega_i\omega_j\rho_{ij}\sigma_i\sigma_j$$

- The standard deviation of the portfolio P is:

$$\sigma_p = \sqrt{\sigma_p^2}$$

Risk reduction in equally-weighted portfolios: Independent returns

- Suppose we have an equally weighted portfolio (holding weights 1/N) of N independent stocks (zero correlation)
- The covariance of the portfolio is

$$\sigma_p^2 = \frac{1}{N^2}\sum_{i=1}^{N}\sigma_i^2 \Longrightarrow \frac{1}{N}\left\{\frac{1}{N}\left\{\sum_{i=1}^{N}\sigma_i^2\right\}\right\}$$

$$= \frac{1}{N}\begin{bmatrix}\text{average}\\ \text{variance}\end{bmatrix}$$

- As the number of assets increase, the risk is diversified away. (The insurance principle)

Risk Reduction in Equally-weighted Portfolios: The General Case

- Suppose we have an equally weighted portfolio (holding weights 1/N) of N independent stocks
- The variance of the portfolio is

$$\sigma_p^2 = \frac{1}{N^2}\sum_{i=1}^{N}\sigma_i^2 + \frac{2}{N^2}\sum_{i=1}^{N}\sum_{j>i}^{N}\mathrm{cov}(R_i, R_j)$$

$\rho_{ij}\ \sigma_i\ \sigma_j$

$$= \frac{1}{N}\left[\frac{1}{N}\sum_{i=1}^{N}\sigma_i^2\right] + \left(1-\frac{1}{N}\right)\left[\frac{1}{N(N-1)/2}\sum_{i=1}^{N}\sum_{j>i}^{N}\mathrm{cov}(R_i, R_j)\right]$$

$$= \frac{1}{N}\begin{bmatrix}\text{Average}\\ \text{Variance}\end{bmatrix} + \left(1-\frac{1}{N}\right)\begin{bmatrix}\text{Average}\\ \text{Covariance}\end{bmatrix}$$

- What happens to the two sources of portfolio variance as $N \to \infty$?

Risk Reduction in Equally-weighted Portfolios: The General Case

- What happens when N goes to infinity?

$$\lim_{N \to \infty} \frac{1}{N} \to 0$$

$$\lim_{N \to \infty} \left(1 - \frac{1}{N}\right) = 1$$

- Variance of the portfolio return
  - Average covariance of returns
- Risk of portfolio --> non-diversifiable risk

Key Lessons

- The key lesson is that the variance or the volatility of an individual asset is not a good indicator of its riskiness! The only thing that matters is its covariance with other assets

- Even if the asset has a low return and high standard deviation, it may be a great asset, when it has low or negative correlation with all other assets in the market.

Classification of Risk

- Part that *cannot* be diversified away
  - "covariance risk", "systematic risk" or "non-diversifiable risk"
  - E.g market risk, macroeconomic risk, industry risk

- Part that *can* be diversified away (in a large portfolio)
  - "variance risk," "non-systematic risk," "diversifiable risk," "idiosyncratic risk," or "unique risk"
  - E.g. Individual company news

Systematic vs. Idiosyncratic risk

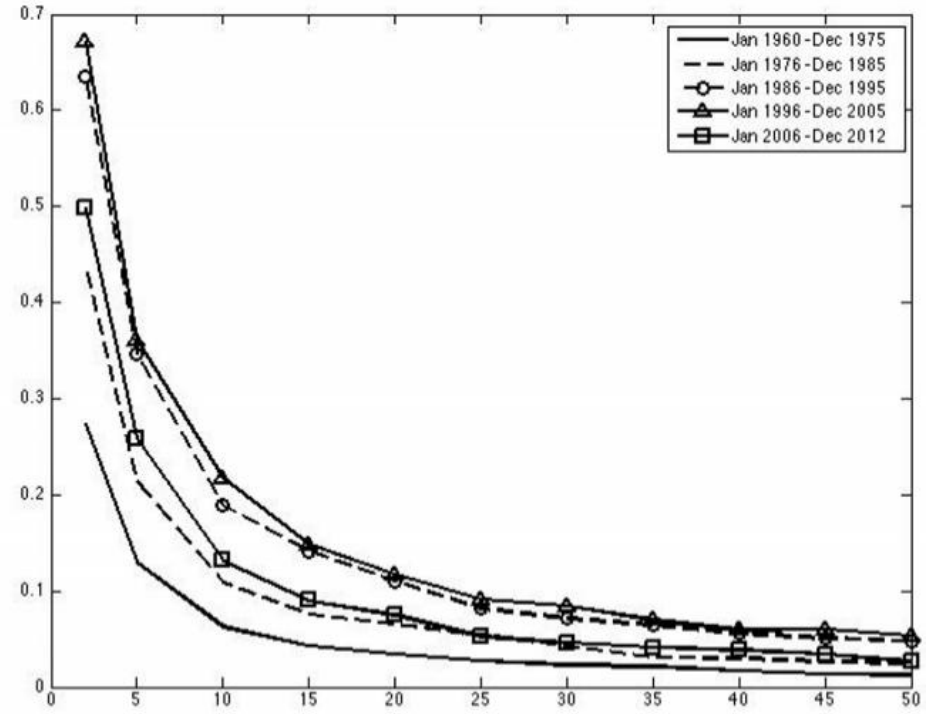
- When held in a portfolio some of the risk of a stock disappears
- Or, the risk contribution the stock makes to the portfolio is LESS than the risk of the stock if held in isolation:

$$\begin{pmatrix}\text{Total risk in}\\ \text{a stock}\end{pmatrix}=\begin{pmatrix}\text{Systematic}\\ \text{risk}\end{pmatrix}+\begin{pmatrix}\text{Idiosyncratic}\\ \text{risk}\end{pmatrix}$$

- **Investors need to be compensated for holding which kind of risk?**

Percent Risk Reduction

- What is the percentage reduction in risk we should expect from adding stocks to our portfolio?
- The below graph shows the incremental standard deviation (on y axis) of a portfolio of N stocks (on x axis) over a fully diversified portfolio



Implementation Issues with Markowitz Model

- With N stocks, one needs:
  - N estimates of expected returns
  - N estimates of variance
  - N(N-1)/2 estimates of correlation between all pairs of returns
  - Not every set of estimates is internally consistent
- For N=500, this amounts to 125,750 parameters that need to be estimated
- **Solution 1**, Index models: all co-movement of returns is captured by one common factor ("the economy")
- **Solution 2**, CAPM : implied expected returns in equilibrium

The Capital Asset Pricing Model (The CAPM)

- The CAPM is an equilibrium model that
  - Predicts the relationship between risk and expected return and suggests optimal portfolio choices
  - Underlies much of modern finance theory and much of real-world financial decision making

- Derived using Markowitz's principles of portfolio theory, with additional simplifying assumptions

- Developed by Sharpe, Lintner and Mossin. William Sharpe won the Nobel Prize in 1990

What is CAPM about?

- Key questions:
  - What portfolios should investors hold *in equilibrium*?
  - What is the *equilibrium required return*, E(R), or the *equilibrium price* of an individual stock?

- Answer: CAPM
  - Identify the tangency portfolio in equilibrium
  - Hence, identify investors' portfolios
  - Derive equilibrium returns (and hence prices)

- The expected return on any asset depends on its covariance with the market portfolio.

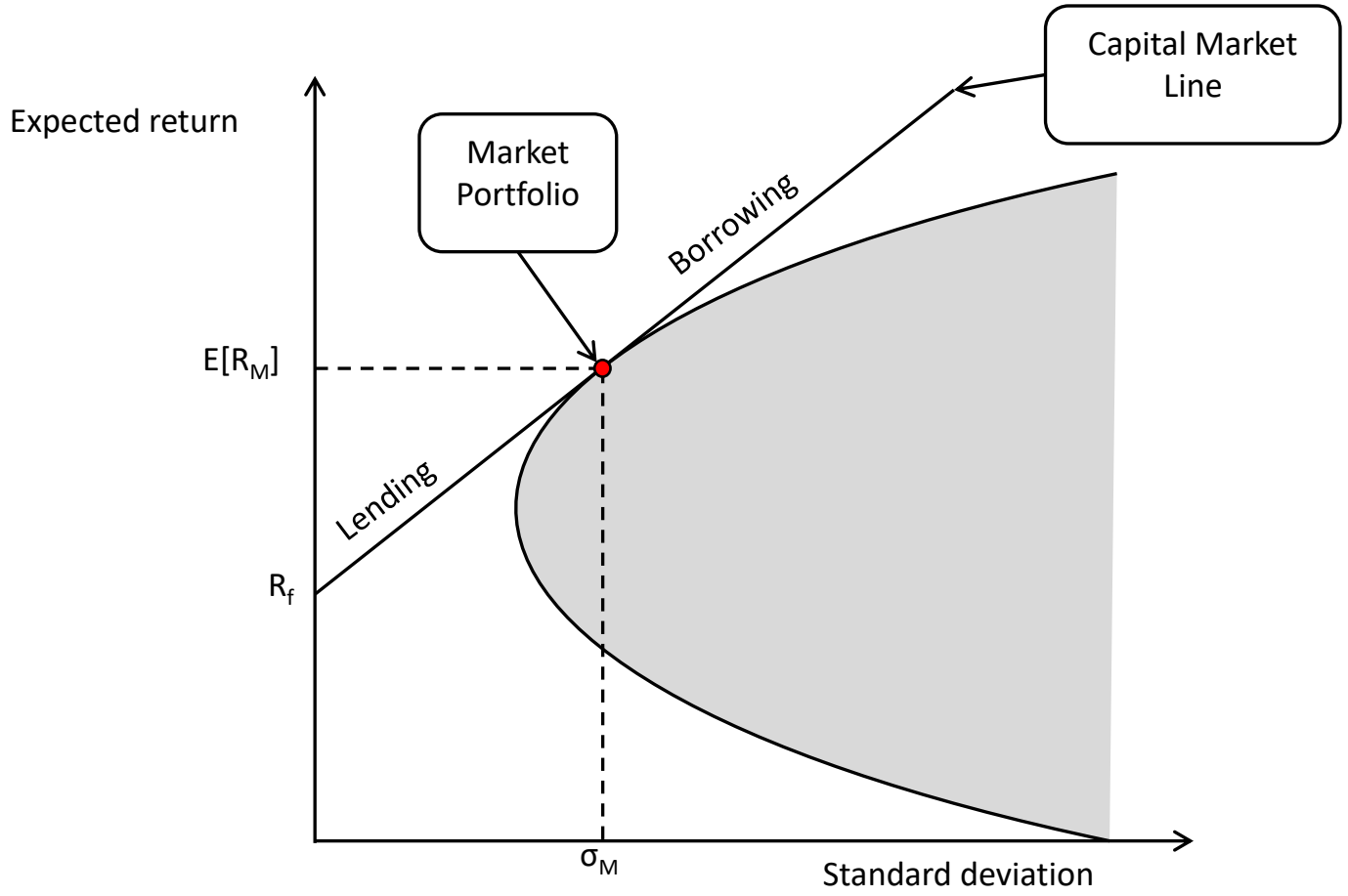
CAPM Assumptions

- Investors are rational mean-variance optimizers with homogeneous expectations or everyone has same information
- The market is in a competitive equilibrium
- Common single period investment horizon
- All assets are tradable
- No transaction costs, no taxes
- Unlimited borrowing at risk-free rate
- Some assumptions can be relaxed and CAPM still holds

Competitive Equilibrium

- What is a competitive equilibrium?
- In economics and finance an equilibrium is a state in which:
  - Prices are such that markets clear. In our setting, supply of an asset= demand of an asset.
  - Every investor is happy with their portfolio at the market prices, with no incentive to change their investment decision. That is, the equilibrium is a "stable" state.

The E(R)-SD Frontier and the Capital Market Line



What is the Equilibrium Tangency Portfolio?

- Recall from portfolio theory,
  - All investors should have a (positive or negative) fraction of their wealth invested in the risk-free security, and
  - The rest of their wealth is invested in the tangency portfolio
  - The *tangency portfolio* is the same for all investors (homogeneous expectations)

- In equilibrium, supply = demand, so:
  - The tangency portfolio must be the portfolio of all existing risky assets, the market portfolio!!

What is the Equilibrium Tangency Portfolio?

- For every investor who overweights an asset relative to the market portfolio ($w_{ip} > w_{iM}$), another investor must underweight it ($w_{ip} < w_{iM}$)
  - The average investor (average dollar) must be invested in the market portfolio
- This is why the market portfolio is a popular benchmark
  - Common proxy: the S&P 500 (~90% of total market cap)

The Market Portfolio

- p_i = price of one share of risky asset *i*
- n_i = number of shares outstanding for risky asset *i*
- Market Portfolio = portfolio in which each risky asset *i* has the following weight

$$\frac{p_i n_i}{\sum_i^N p_i n_i} = \frac{Market\ cap\ of\ Security\ i}{Total\ Market\ Capitalization}$$

- In words-the market portfolio is the portfolio consisting of all assets (everything!)

The Capital Market Line (CML)

- In equilibrium, the tangency portfolio is the market portfolio
- The market portfolio's CAL is called the Capital Market Line (CML)
- The CML gives the risk-return combinations achieved by forming portfolios from the risk-free security and the market portfolio

$$E[R_p] = R_f + \left(\frac{E(R_M) - R_f}{\sigma_M}\right)\sigma_p$$

- Sharpe Ratio= "Price of Risk"

Return of Market Portfolio

$$E(R_p) = R_f + \left(\frac{[E(R_M) - R_f]}{\sigma_M} \right) \sigma_p$$

- In equilibrium, the return of the market depends on the riskiness (SD) of the market and the price of the risk

- How should we think about individual stock's return?

The Required Return on Individual Stocks

- CAPM is most famous for its prediction about the expected returns of individual securities.
- In equilibrium, expected returns (or, equivalently, prices) must be such that investors are satisfied with their portfolio holdings.
- Suppose you hold the market portfolio (the tangency portfolio). You decide to sell a small amount of the risk-free asset and add a small amount ∂w_i of the security *i*.
- What must be the incremental risk-return tradeoff this new security gives you?
- Can it be higher than the market portfolio? Can it be lower than the market portfolio?

The Required Return on Individual Stocks

- CAPM is most famous for its prediction concerning the relationship between risk and expected return for individual securities
- What is each security's contribution to the riskiness of the portfolio? (think back to the class about diversification)
- The extra return is proportional to the risk contribution of that security to the overall market
- This relationship must hold for every security. Why?

Risk Contribution of a Security

- GE's contribution to the portfolio risk = $W_{GE} * Cov(R_M, R_{GE})$
- Similarly, GE's contribution to the return of the portfolio = $W_{GE} * E(R_{GE} - R_f)$
- Hence, the reward to risk ratio for investment in GE is

$$\frac{GE's\ contribution\ to\ the\ risk\ premium}{GE's\ Contribution\ to\ the\ variance} = \frac{W_{GE}E(R_{GE} - R_f)}{W_{GE}Cov(R_{GE}, R_M)} = \frac{E(R_{GE} - R_f)}{Cov(R_{GE}, R_M)}$$

- The reward to risk ratio of the market is

$$\frac{Market's\ risk\ premium}{Market's\ variance} = \frac{E(R_M - R_f)}{Cov(R_M, R_M)} = \frac{E(R_M - R_f)}{\sigma_M^2}$$

Reward to Risk Ratio

- In equilibrium, all investments should offer the same reward to risk ratio. If not, the prices will adjust.
- This implies:

$$\frac{E(R_{GE} - R_f)}{Cov(R_{GE}, R_M)} = \frac{E(R_M - R_f)}{\sigma_M^2}$$

- Rewriting:

$$E\left(R_{GE} - R_f\right) = \frac{Cov(R_{GE}, R_M)}{\sigma_M^2} \times E(R_M - R_f)$$

The Main Equation of CAPM

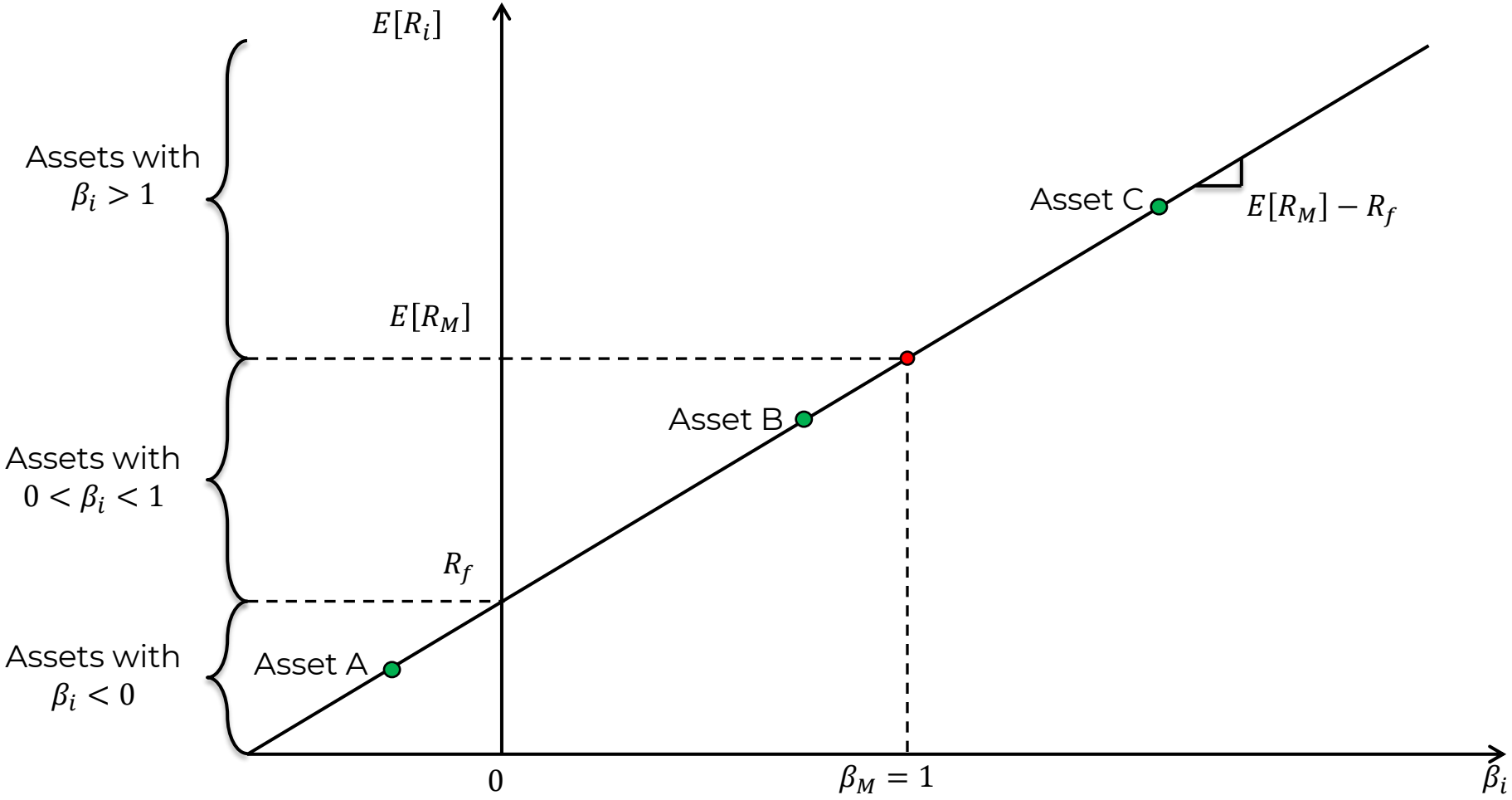
$$E(R_i) = R_f + \frac{Cov(R_i, R_M)}{\sigma_M^2} \times E(R_m - R_f)$$

- Here $\frac{COV(R_i,R_M)}{\sigma_M^2}$ is known as β_i. It tells you how much R_i increases on average when $E(R_m - R_f)$ increases. So

$$E(R_i) = R_f + \beta_i \times E(R_m - R_f)$$

- Intuition: investors are compensated for holding stocks with higher market risk.
- This goes back to the risk-return tradeoffs we've been discussing throughout, and one of our starting axioms: no free lunch in finance.

Security Market Line (SML)



Securities Market Line

- The model predicts that expected excess return of a security is linear in its beta:

$$E[R_i] = R_f + \beta_i E[R_M - R_f]$$

- This linear relation is the Security Market Line (SML).
- The beta measures the security's sensitivity to market movements:

$$\beta_i = \frac{Cov(R_i, R_M)}{Var(R_M)}$$

Practice Questions

- What is the beta of the risk-free security?
- What is the beta of the market?
- What must be the β_i of a portfolio with E(R) = 18%, a risk-free rate of 6% and a market return of 14%.
  - First question: Is it larger or smaller than 1?

Practice Questions

- Let's use the CAPM to figure out the expected returns of these stocks.

	X	Y	Z	Market	Risk-free
Expected Return	??	??	??	10%	2%
SD	25%	25%	25%	20%	
Correlation	0.0	0.8	-0.4		
Beta	??	??	??		

Summary

- SML: $E(R_i) = R_f + \beta_i [E(R_M) - R_f]$
- Stock *i*'s systematic or market risk is β_i
- Investors are compensated for holding systematic risk in the form of higher returns.
- The size of the compensation depends on the *Equilibrium Risk Premium*, $E(R_M) - R_f$.
- The equilibrium risk premium is **increasing** in:
  - The variance of the market portfolio (volatile times)
  - The degree of risk aversion of *average* investor