COMILLAS
UNIVERSIDAD PONTIFICIA
ICAI ICADE CIHS

Máster Universitario en Gestión de Riesgos Financieros

Principios de Identificación y Gestión de Riesgos

Riesgo de tipo de interés

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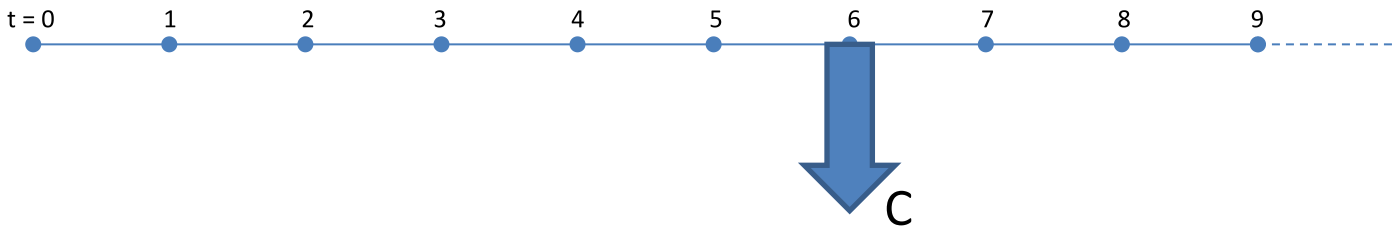
5 de noviembre de 2025

¿De qué vamos a hablar?

- **Matemática Financiera**
  - El Valor Actual Neto (VAN)
  - La Tasa Interna de Retorno (TIR)
- **Renta fija**
  - ¿Qué es un bono?
  - La curva de tipos
- **Duración**
  - Duración normal y modificada
  - Convexidad
- **Riesgos y renta fija**
  - Riesgo de tipo de interés - Precio
  - Riesgo de tipo de interés - Reinversión
  - Riesgo de crédito

Valor Actual Neto



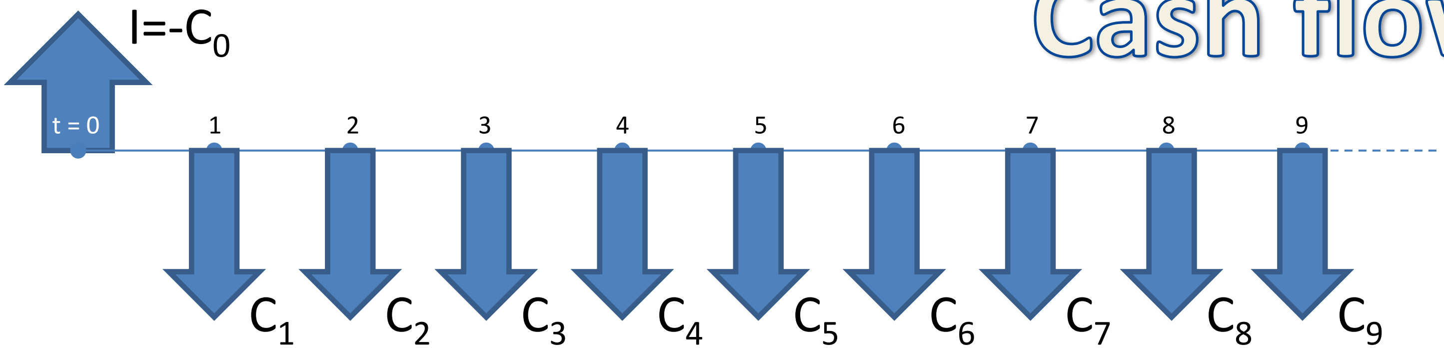
How much must I invest at a rate of *i%* payable annually to obtain C after 6 years?

Present value give us the present day worth of future amounts. What is the value now of C within 6 years considering a rate of i%?

$$I = \frac{C}{(1+i)^6} \Rightarrow PV = \frac{C}{(1+i)^6}$$

Valor Actual Neto

Cash flow



Net Present Value (NPV)

$$NPV = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \ldots + \frac{C_9}{(1+r)^9}$$

Valor Actual Neto

Calculate **NPV** at **10%** of following invest:

t	0	1	2	3	4	5	6	7	8	9
Q	-10.000	2.200	2.200	2.200	2.900	2.200	2.200	2.000	2.000	2.000

Solution: We can divide the rent into four components:

t	0	1	2	3	4	5	6	7	8	9
Q0	-10.000	-	-	-	-	-	-	-	-	-
Q1	-	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
Q2	-	200	200	200	200	200	200	-	-	-
Q3	-	-	-	-	700	-	-	-	-	-

$$VAN = Q_0 + Q_1 + Q_2 + Q_3 = -10.000 + 1.000\sum_{n=1}^{9}\frac{1}{(1+r)^n} + 200\sum_{n=1}^{6}\frac{1}{(1+r)^n} + \frac{700}{(1+r)^4}$$

$$VAN = -10.000 + 2.000\frac{1-1,1^{-9}}{0,1} + 200\frac{1-1,1^{-6}}{0,1} + \frac{700}{1,1^4} = -10.000 + 11.518,05 + 871,05 + 478,11 = 2.867,21$$

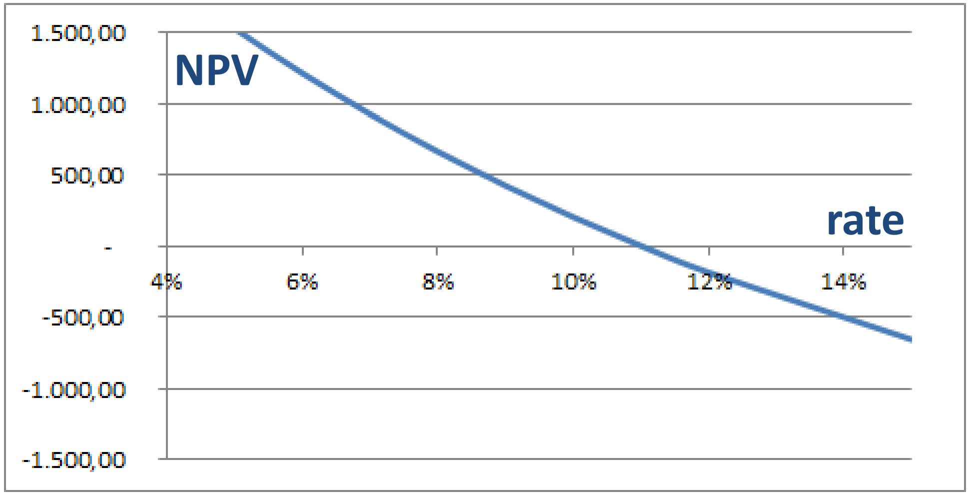
TIR – Tasa Interna de Retorno

Calculate NPV at **8%** and **12%** of following amounts **by using excel**:

	0	1	2	3	4	5	6	7	8	9	10
C	-1.200	-1.200	-1.200	-1.200	1.200	1.200	1.200	1.200	1.200	1.200	1.200

NPV (8%) = 667,06 **NPV (12%)** = -184,13

Repeat the same exercise with **5%** and with **10%**...



NPV (11,01%) = 0

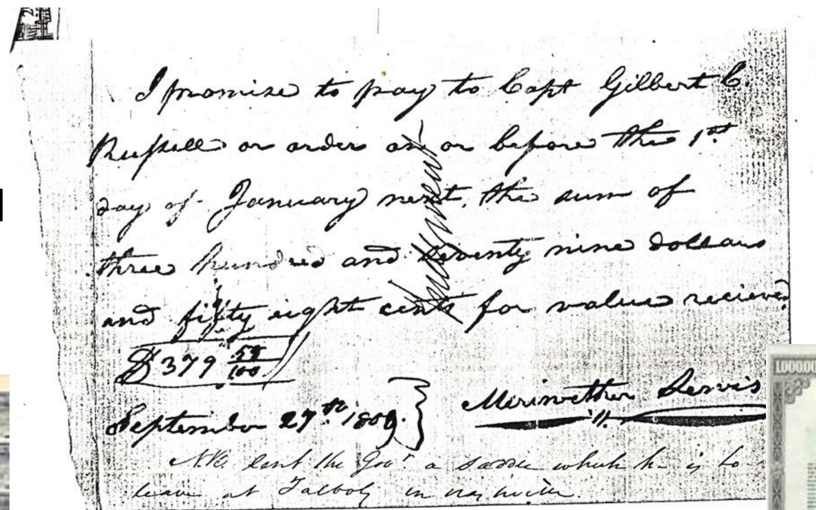
Internal Rate of Return (IRR)
11,01%

Renta fija

¿Qué es un bono?

IOU note (I owe you)

- The issuer: borrower
- Investor: lender
- Face value: amount borrowed
- Maturity date: term
- Coupons: fixed at issuance


I promise to pay to Capt Gilbert C. Russell or order at or before the 1st day of January next, the sum of three hundred and seventy nine dollars and fifty eight cents for value received

$379 58/100

September 27th 1809 Meriwether Lewis

N.B. lent the Gov. a saddle which he is to leave at Talbots in Nashville


The Malden & Melrose Railroad Co.

[$15] Will pay the Bearer, at the Office of their Treasurer, in Boston, FIFTEEN DOLLARS, on the 1st day of April, 1863, for six months' interest on Bond No. [illegible] dated [illegible] 1st, 1860.

Treasurer.

¿Qué es un bono?

Principales características

1. **Issuer**
   - US Treasury/Government
   - States, municipalities, and agencies
   - Corporations
   - Foreign governments (sovereign bonds)
2. **Term** (maturity)
   - Short –T-bills, CD's, commercial paper
   - Long—T-bonds, corporate bonds
3. **Price vs. par value (face value)**
   - Par bond
   - Discount bond
   - Premium bond
4. **Coupon**
   - Without coupons: zero coupon bond
   - Coupon rate: total annual interest payment per € face value
   - Frequency (usually semi-annual)
   - Fixed or variable (floaters)
   - Nominal or inflation-indexed (TIPS)
5. **Currency**
   - Yankee bonds, Samurai bonds
   - Eurobonds
6. **Credit risk**
   - Risk-free
   - Defaultable
7. **Seniority and security**
   - Senior, senior subordinated, junior, etc.
   - Secured by property, income stream, etc.
8. **Covenants**
   - Restrictions on additional issues, dividends, and other corporate actions
9. **Option provisions**
   - Callability: Issuer has the right to pay back the loan before maturity
   - Putability: Bondholder has the right to demand payment of the loan before maturity
   - Convertibility: Bondholder has the right to exchange the bond for stocks of the issuer

Renta fija - Bonos

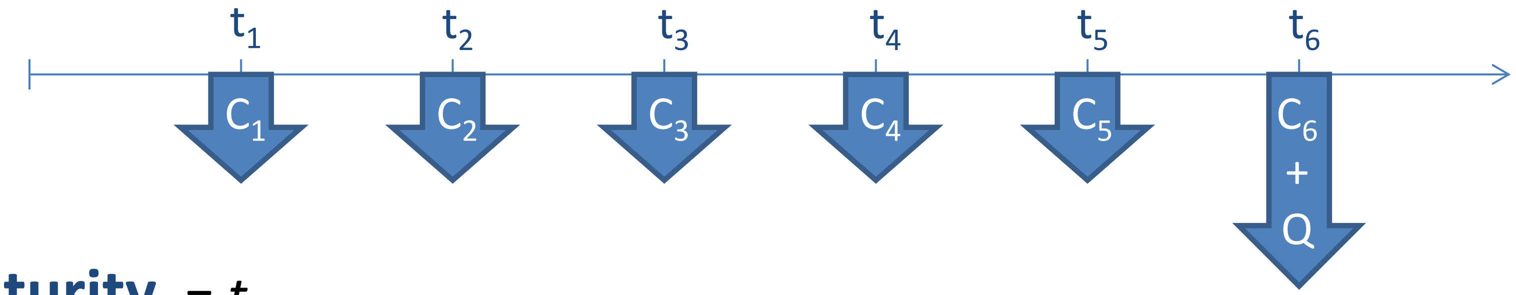
Main risks born by the investor:

- Credit risk or default risk
- Interest rate risk: the return to the investor is NOT fixed, since it is influenced by the evolution of the instrument's market value, which is in turn related to the evolution of interest rates
- High inflation rates turn Fixed Income a less attractive investment

Main types of Bonds:

- Bills (<1year), notes (1<maturity<10years) and bonds (10 years<maturity)
- Indexed
- Profit-Sharing
- Convertibles, etc…

Renta fija
Vencimiento, precio y duración



Maturity $= t_6$

Price $$= VA(C_1) + VA(C_2) + \ldots + VA(C_6 + Q) = \frac{C_1}{(1+r)^{t_1}} + \frac{C_2}{(1+r)^{t_2}} + \ldots + \frac{C_6 + Q}{(1+r)^{t_6}}$$

Duration $$= \frac{t_1 \cdot VA(C_1) + t_2 \cdot VA(C_2) + \ldots + t_6 \cdot VA(C_6 + Q)}{VA(C_1) + VA(C_2) + \ldots + VA(C_6 + Q)} = \frac{t_1 \cdot \frac{C_1}{(1+r)^{t_1}} + t_2 \cdot \frac{C_2}{(1+r)^{t_2}} + \ldots + t_6 \cdot \frac{C_6 + Q}{(1+r)^{t_6}}}{\frac{C_1}{(1+r)^{t_1}} + \frac{C_2}{(1+r)^{t_2}} + \ldots + \frac{C_6 + Q}{(1+r)^{t_6}}}$$

Renta fija – Yield to maturity

Yield to maturity

- The price of a two year bond with payments C_1 and C_2 would be : $PV = C_1 / (1+r_1) + C_2 / (1+r_2)^2$
- The first period's coupon is discounted at today's **one-period spot rate** and the second period´s coupon is discounted at today´s **two-period spot rate**. The series of spot rates r_1, r_2, etc… is the Term Structure of Interest Rates (TSIR).
- Rather than discounting each of the coupons at a different rate, we could find a single rate that could give us the same value. This is the IRR of the bond, also known as the Yield to Maturity. **Yields to Maturity do not determine bond prices, it is the other way around**.

The Yield to Maturity is just a simplified measure that assumes a flat interest rate curve with parallel upward and downward movements

So, the formula which relates the price of a bond to its YTM is the following:

$\boxed{P = \Sigma C_i / (1+r)^I}$ where P stands for the Price, C stands for the Coupon and r stands for the YTM.

Duración – Sensibilidad al tipo de interés

- **First order effect:** Bond prices and interest rates are negatively related

Calculate price of a one year zero coupon bond and a two years coupon bond at YTM of 5%, 6% and 7%.

$$P_{ZCC1} = \frac{100}{1 + YTM}$$

$$P_{ZCC2} = \frac{100}{(1 + YTM)^2}$$

YTM	1-year	2-year
5%	95,24	90,70
6%	94,34	89,00
7%	93,46	87,34

- **Maturity matters:** Prices of long-term bonds are more sensitive to interest rate changes than short-term bonds

% change 1y 6%→7%: (93.46 – 94.34)/94.34 = -0.93%
% change 2y 6%→7%: (87.34 – 89.00)/89.00 = -1.86%

- **Convexity:** An increase in a bond's YTM results in a smaller price decline than the price gain associated with a decrease of equal magnitude in the YTM

Negative convex relation between YTM and price:
Price change 2-year ZCB 5%→6%: 89.00 – 90.70 = -1.70
Price change 2-year ZCB 6%→7%: 87.34 – 89.00 = -1.66

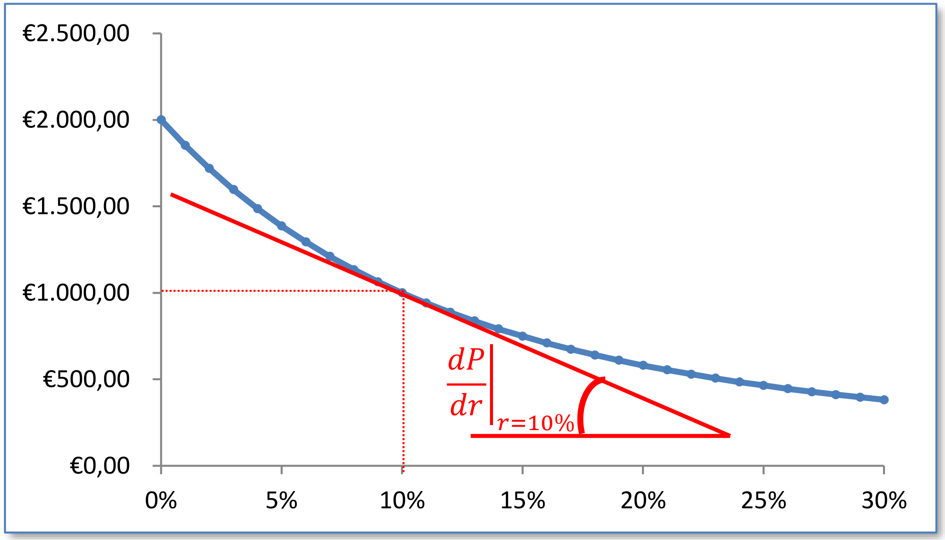
Duración – Sensibilidad al tipo de interés

Price / yield relationship

A Bond is issue without discount, with maturity of 10 years, facial of 1.000€ and ten coupons of 100€ to be paid yearly. Calculate relationship between price and interest rate.

Time	1	2	3	4	5	6	7	8	9	10
Payments	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	1.100,00

$$P=\frac{100}{1+r}+\frac{100}{(1+r)^2}+\frac{100}{(1+r)^3}+\ldots+\frac{100}{(1+r)^8}+\frac{100}{(1+r)^9}+\frac{1.100}{(1+r)^{10}}$$



0%	2.000,00 €
1%	1.852,42 €
2%	1.718,61 €
3%	1.597,11 €
4%	1.486,65 €
5%	1.386,09 €
6%	1.294,40 €
7%	1.210,71 €
8%	1.134,20 €
9%	1.064,18 €
10%	1.000,00 €
11%	941,11 €
12%	887,00 €
13%	837,21 €
14%	791,36 €

Duración – Sensibilidad al tipo de interés

Derivation

The equilibrium price (P) of a financial asset, knowing the cash flows (CFi) generated each year (i), during a determined period of time (T), discounted at a rate (y) is described by following equation:

$$P = \frac{CF_1}{(1+y)^1} + \frac{CF_2}{(1+y)^2} + \ldots + \frac{CF_T}{(1+y)^T}$$

If we derivate price formula we got:

$$\frac{dP}{dy} = -1\frac{CF_1}{(1+y)^2} - 2\frac{CF_2}{(1+y)^3} - \ldots - T\frac{CF_T}{(1+y)^{T+1}}$$

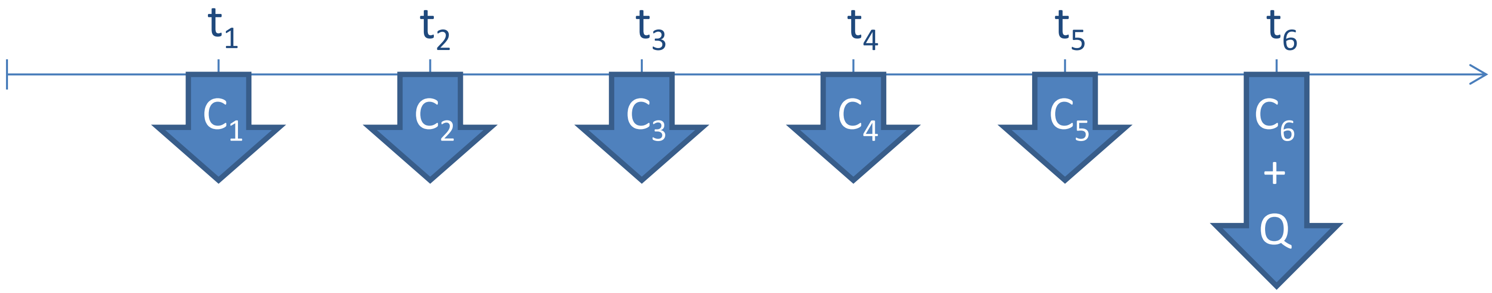
The **price sensitivity** is linked to the **average time** you have to wait for your payments, provided the weights are the contributions to the price of the bond.

And we can get following equation:

$$-\frac{dP}{dy}\frac{1+y}{P} = \underbrace{\left(1\frac{CF_1}{(1+y)^1} + 2\frac{CF_2}{(1+y)^2} + \ldots + T\frac{CF_T}{(1+y)^T}\right)\frac{1}{P}}_{\text{Duration (D)}}$$

Duración – Sensibilidad al tipo de interés

Duration (or Macaulay duration)



Duration $= \dfrac{t_1 \cdot VA(C_1) + t_2 \cdot VA(C_2) + \ldots + t_6 \cdot VA(C_6+Q)}{VA(C_1) + VA(C_2) + \ldots + VA(C_6+Q)} = \dfrac{t_1 \cdot \dfrac{C_1}{(1+r)^{t_1}} + t_2 \cdot \dfrac{C_2}{(1+r)^{t_2}} + \ldots + t_6 \cdot \dfrac{C_6+Q}{(1+r)^{t_6}}}{\dfrac{C_1}{(1+r)^{t_1}} + \dfrac{C_2}{(1+r)^{t_2}} + \ldots + \dfrac{C_6+Q}{(1+r)^{t_6}}} =$

$$\textbf{Duration} = \frac{t_1 \cdot \dfrac{C_1}{(1+r)^{t_1}} + t_2 \cdot \dfrac{C_2}{(1+r)^{t_2}} + \ldots + t_6 \cdot \dfrac{C_6+Q}{(1+r)^{t_6}}}{P}$$

- The sensitivity of the price of a bond to changes in the yield.
- Can often be interpreted as the "average" time you have to wait for your payments.

Duración – Sensibilidad al tipo de interés

Duration (or Macaulay duration)

Calculate duration of following cash flow using a discount of 15%:

Tiempo	1	2	3
Flujo de caja	3.000,00	2.000,00	1.000,00

$$duration = \frac{1\cdot\frac{3.000}{1,15}+2\cdot\frac{2.000}{1,15^2}+3\cdot\frac{1.000}{1,15^3}}{\frac{3.000}{1,15}+\frac{2.000}{1,15^2}+\frac{1.000}{1,15^3}} = 1,59 \text{ years}$$

Calculate duration of a **zero coupon bond** with maturity of 10 years and a **yield to maturity** of 5%. Also calculate the issue price and the **discount**. (Nominal=1.000€)

duration = *maturity* = 10 years

$$Price = \frac{1.000}{1,05^{10}} = 613,91$$

$$Discount = \frac{1.000-613,91}{1.000} = 38,61\%$$

Duración – Sensibilidad al tipo de interés

Duration (or Macaulay duration)

A Bond is issue without discount, with maturity of 3 years, facial of 1.000€ and three coupons of 100€ to be paid yearly. Calculate yield to maturity and duration (with IRR)

Time		1	2	3
Debt	- 1.000,00			1.000,00
Coupon		100,00	100,00	100,00
Cash Flow	- 1.000,00	100,00	100,00	1.100,00

$$duration = \frac{1\cdot\frac{100}{1,1} + 2\cdot\frac{100}{1,1^2} + 3\cdot\frac{1.100}{1,1^3}}{\frac{100}{1,1} + \frac{100}{1,1^2} + \frac{1.100}{1,1^3}} = 2,74 \text{ years}$$

$$IRR \Rightarrow -1.000 + \frac{100}{1+r} + \frac{100}{(1+r)^2} + \frac{1.100}{(1+r)^3} = 0 \;\Rightarrow\; IRR = 10\% \;\Rightarrow\; \frac{100}{1,1} + \frac{100}{1,1^2} + \frac{1.100}{1,1^3} = 1.000$$

A Bond is issue without discount, with maturity of 10 years, facial of 1.000€ and ten coupons of 100€ to be paid yearly. Calculate yield to maturity and duration (with IRR)

Time		1	2	3	4	5	6	7	8	9	10
Debt	- 1.000,00										1.000,00
Coupon		100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00
Cash flow	- 1.000,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	1.100,00

	Total										
NPV	1.000,00	90,91	82,64	75,13	68,30	62,09	56,45	51,32	46,65	42,41	424,10
time x NPV	6.759,02	90,91	165,29	225,39	273,21	310,46	338,68	359,21	373,21	381,69	4.240,98
Duration	**6,76**										

Duración – Sensibilidad al tipo de interés

Modified duration

The duration (D) of a bond is defined as minus the elasticity of its price (P) with respect to (1 plus) its YTM (y):

$$D = -\frac{dP}{dy}\frac{1+y}{P} = \sum_{1}^{T} w_t\, t \quad \text{where} \quad w_t = \left(\frac{CF_t}{(1+y)^t}\right) \Big/ P = PV(CF_t)/P$$

For fixed cash flows, duration is equal to the average of the cash-flow times, weighted by their contribution to the present value of the bond.

The price response to a yield change is therefore:

$$\frac{\Delta P}{P} \cong -\underbrace{\frac{D}{1+y}}_{\substack{\text{modified}\\\text{duration}}}\,\Delta y$$

Duración – Sensibilidad al tipo de interés

Modified duration – an example

What is the duration of a 3-year coupon bond with a face value of $1000, a coupon rate of 8%, and a YTM of 10%?

P = 80/(1.10) + 80/(1.10^2)+1080/(1.10^3) = 950.26
w_1 = (80/1.1)/950.26 = 0.077
w_2 = (80/1.1^2)/950.26 = 0.070
w_3 = (1080/1.1^3)/950.26 = 0.854
D = 1*w_1 + 2*w_2 + 3* w_3 = 2.78 (years BUT it is actually unitless)

If the YTM changes to 10.1%, what would be the (relative) change in price?

Let's use our formula: Δy=0.0010=0.10%=10bp [basis points]
ΔP/P≈-{D/(1+y)}Δy=-(2.77/1.10)*0.0010=-0.0025=-0.25%
ΔP = P(-0.25%) = 950.26(-0.25%)=-$2.40
Good approx.? Price at 10.1% P=947.87, price drop is $2.39

Duración – Sensibilidad al tipo de interés

Duration – Useful facts

- What is the duration of zero-coupon bond?
- What must be true for the duration of a coupon bond?
- What happens to the duration of a coupon bond if (all else equal) the coupon rate increases?
- What happens to the duration of the bond if (all else equal) the YTM increases?
- Factors affecting duration
  - Maturity (+)
  - Coupon rate (-)
  - YTM (-)
- The duration of a portfolio is the weighted average of the durations of the constituents: $D_p = \sum_i w_i D_i$
- What is the duration of a perpetuity?

    Let's apply the definition
    P = C/y => dP/dy = -C/y^2 ⍰ D = (dP/dy) * -(1+y)/P = (C/y^2) * (1+y)/(C/y) = (1+y)/y
    For example: 8% YTM: 1.08/.08 = 13.5 years duration.

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Duración – Sensibilidad al tipo de interés

Convexity

- The sensitivity of price with respect to yield is approximated by a linear function when using duration
- The relation is really non-linear (convex)
  - When yields decline, the price increase in the bond is underestimated by the simple duration formula
  - When yields increase, the price decline in the bond is overestimated by the simple duration formula
  - A convexity term corrects the problem

If we compare different zero coupon bonds with different YTM:

		5,00%	6,00%	Δprice	ΔΔprice
Comparing 1y -2y	1y	95,24	94,34	-0,94%	199,06%
	2y	90,70	89,00	-1,88%	
Comparing 29y-30y	29y	24,29	18,46	-24,03%	102,98%
	30y	23,14	17,41	-24,75%	
Comparing 20y - 30y	20y	37,69	31,18	-17,27%	143,32%
	30y	23,14	17,41	-24,75%	

Regarding interest rates changes, there is a lot of change in short maturity bonds while longer maturity ones behavior is similar.

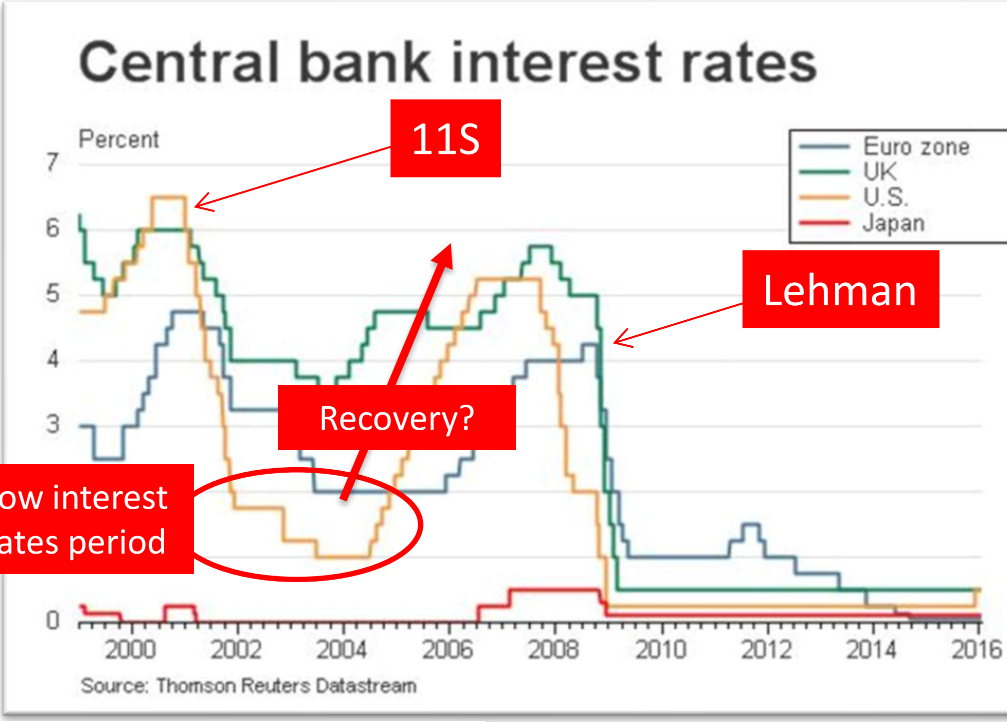
Referencias de tipos de interés

What about interest rates?

Central Banks (CB) interest rate.
CB lend money to other banks.

- **Currency**
- **CB reserves**
- **Demands deposits and equivalent**
- **Saving deposits** and equivalent
- **Large time deposits and equivalent**

EONIA - EURIBOR Banks lend money to each other. EURIBOR is the daily average rate of these loans.



Three banks accused of rigging Euribor

HSBC, JP Morgan and Credit Agricole under investigation by European Commission over allegations of cartel behaviour

Link - The telegraph (2014)

Temporal Structure of Interest Rates (TSIR) Governments need to be financed. They sell debt and will pay back with difference maturities.

Renta fija – La curva de tipos

The Yield Curve shows the relationship between return and time to maturity It is widely used to value financial assets.

The most widely used Yield Curve is the one built with public debt securities, given their:

- Risk-free characteristics
- Higher liquidity

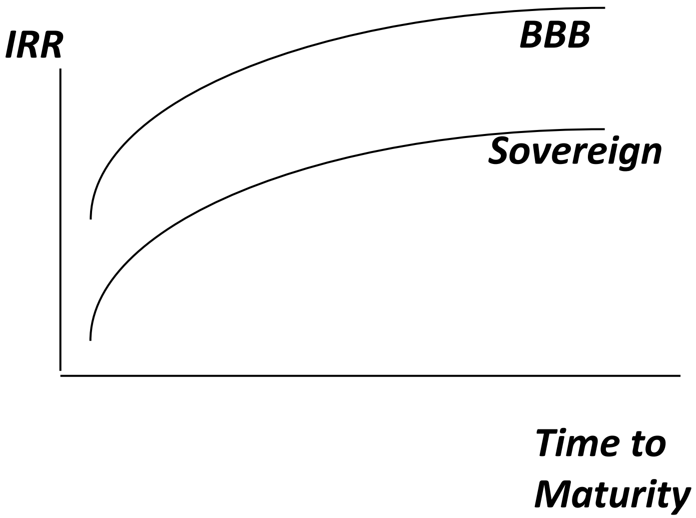
The most liquid and more frequently traded issues in the secondary market are used to build the curve.

The most usual shape of the Yield Curve is a growing curve.

The shorter terms are more volatile and more highly influenced by monetary policy decisions. Longer terms are more stable and influenced by economic expectations

Implicit interest rates are derived from the Yield Curve…

We have seen this before
Just recap



Renta fija – La curva de tipos

Term implicit interest rates are derived from the Yield Curve

- **Spot interest rates:** rates currently required by investors
- **Term interest rate:** rate required for a certain time period in the future. Term rates can be derived from the Yield Curve

Prior to the calculation of implicit term rates, the zero-coupon curve must be obtained. The zero-coupon curve provides us with the spot rates necessary to obtain the term rates thereafter

- Every bond can be broken down into as many zero-coupon bonds as the number of coupons the bond has. A 10-yr €1,000 bond with 6% annual coupons can be broken down into nine €60 zero-coupon bonds and one €1,060 zero-coupon.

- Investing in a one-year bond at r_1 and then reinvesting at ${}_1r_2$ (the spot rate at time 1 on a bond maturing at time 2) yields the following Payoff: $(1+ r_1) * (1 + {}_1r_2)$
- Investing in a two-year bond at the two-year spot rate r_2 yields the following payoff: $(1 + r_2)^2$

The second way of investing can be reinterpreted as investing for 1 year at r_1 and for the second year at a forward rate f_2. f_2 is implicit in the two-year spot rate r_2

$$\mathbf{(1 + r_2)^2 = (1+ r_1) * (1 + f_2)}$$

Renta fija – La curva de tipos

http://www.bde.es/webbde/es/secciones/informes/banota/boletin.html

E M I S I O N	NÚMERO OPERACS	IMPORTE CONTRAT	PRECIO (EX-CUPON) MEDIO	MAXIMO	MINIMO	RENDTO. INTERNO	ANTERIOR PRECIO MEDIO
ES00000128E2 O EST 3.45 30.07.66	2	142,02	122,28	125,88	118,69	2,64	119,059 (15/09/2016)
ES00000124I2 B EST 2.10 30.04.17	1	11,00	101,45	101,4			
ES00000124B7 B EST 3.75 31.10.18	1	0,07	108,33	108,3			
ES(				100,7			
ES(				107,3			
ES(				113,60	113,60	- 0,05	113,362 (15/09/2016)
ES(				114,38	114,38	0,01	114,286 (14/09/2016)
ES(							
ES(							
ES(							
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ES(							
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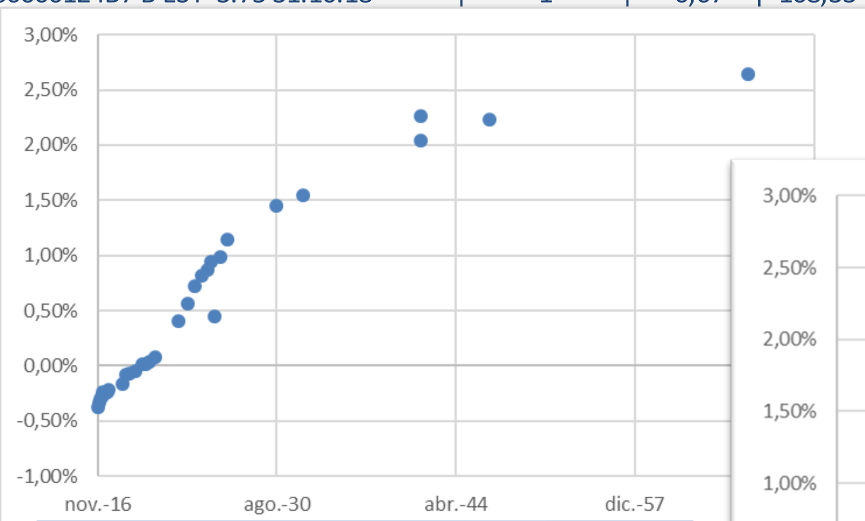
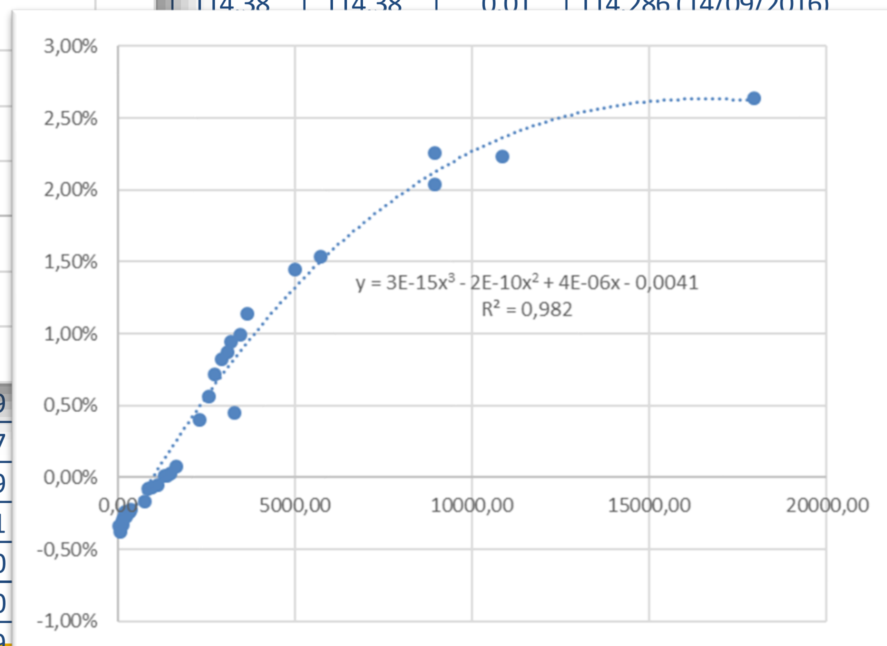
ES(							
ES(							
ES000		39					
ES000		87					
ES000		69					
ES000		01					
ES000		00					
ES000		00					
ES000		29					

Features of government treasure TSIR:
- no credit risk (in theory)
- no liquidity problems





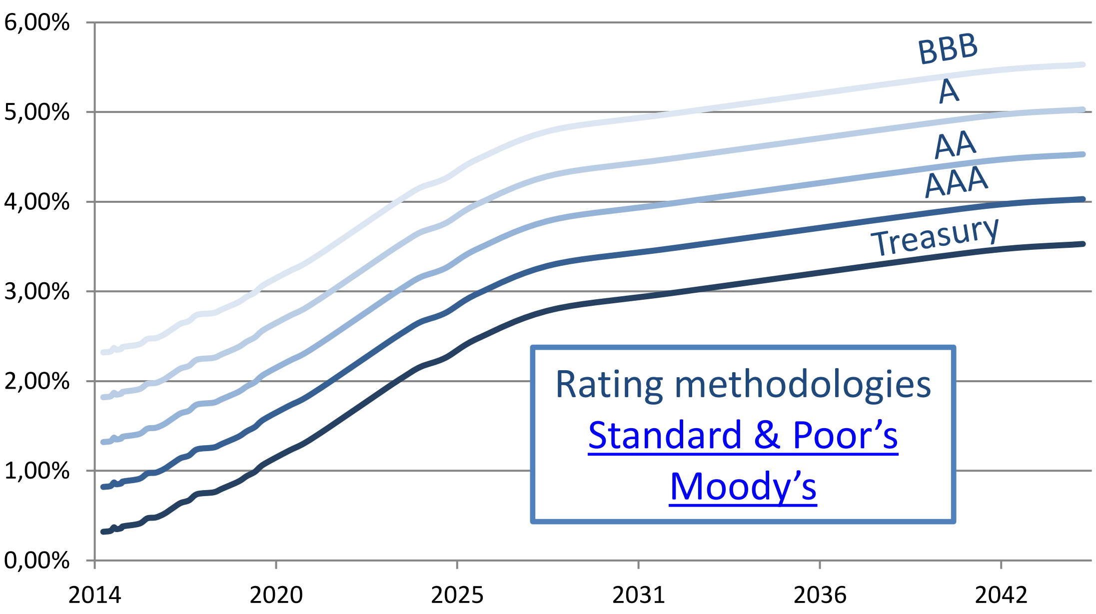
Letras del Estado (Zero coupon)
Maturity: 3, 6, 12 and 18 months

Bonos del Estado (Yearly coupon)
Maturity: 3 and 5 years

Obligaciones del Estado (yearly coupon)
Maturity: 10, 15 and 30 years.

Renta fija – La curva de tipos

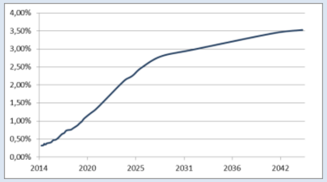
There is a different TSIR each day, and not all kind of assets fit in the same TSIR.



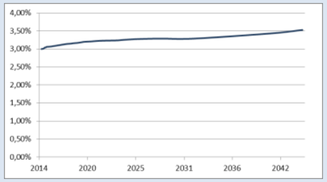
Rating methodologies
[Standard & Poor's]
[Moody's]

Types of curves

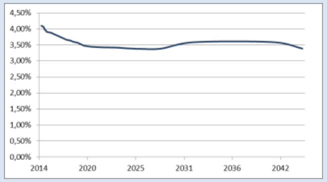
Normal curve (increasing)



Flat curve



Decreasing

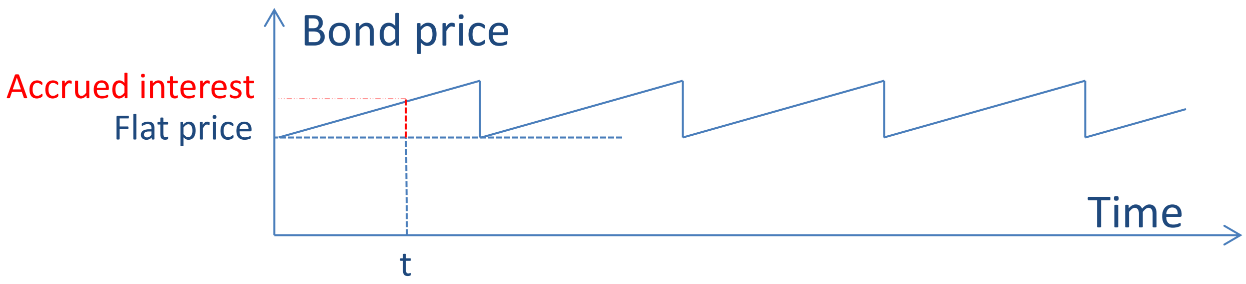


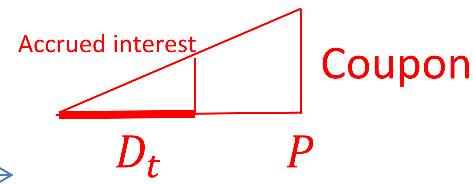
Dynamic yield – curve vs SP500

http://stockcharts.com/freecharts/yieldcurve.php

Otros conceptos – Cupón corrido

http://www.bde.es/webbde/es/secciones/informes/banota/boletin.html





Bond price (t) = Flat price + Accrued interest

$$\text{Accrued interest} = \text{Coupon} \cdot \frac{D_t}{P}$$

E M I S I O N	NUMERO OPERACS	IMPORTE CONTRATADO	PRECIO (EX-CUPON)			RENDTO. INTERNO MEDIO	ANTERIOR PRECIO MEDIO (FECHA)
			MEDIO	MAXIMO	MINIMO		
ES00000123T1 B EST 2.75 31.03.15	1	15,00	100,930	100,930	100,930	0,21	100,945 (12/11/2014)
ES00000123P9 B EST 3.75 31.10.15	2	0,01	103,274	103,288	103,260	0,30	103,326 (12/11/2014)
ES00000122X5 B EST 3.25 30.04.16	4	2,01	104,140	104,163	104,130	0,38	104,170 (11/11/2014)
ES00000123W5 B EST 3.30 30.07.16	1	3,70	104,870	104,870	104,870	0,42	104,900 (11/11/2014)
ES00000120J8 O EST 3.80 31.01.17	2	0,03	107,275	107,300	107,250	0,48	107,350 (12/11/2014)
ES00000124I2 B EST 2.10 30.04.17	2	20,00	103,690	103,700	103,680	0,58	103,810 (12/11/2014)

Otros conceptos – Cupón corrido

Clean Price and Dirty Price

The Clean Price is the price of a bond excluding any interest that has accrued since issue or the most recent coupon payment. The Dirty Price is the price of a bond including the accrued interest.

Clean prices are more stable and change for an economic reason. Dirty prices change day to day depending on where the current date is in relation to the coupon dates, in addition to any economic reasons.

Example: €55 annual coupon, payable on October 31st

Bond bought on Sep 8th 2009. In this case, the coupon has been accrued for 308 days

Accrued Coupon = 308*55/360 = €47

Call Provisions. Corporate bonds sometimes include a call option that gives the company – issuer- the right to pay back the debt early.

Seniority. Bonds may be senior claims or subordinated to the senior bonds. If the firm defaults, the senior bonds come first in the pecking order.

Covenants. Covenants are the contract clauses which attempt to reduce the conflicts of interest between debtholders and equityholders.

Riesgo de tipo de interés - Precio

A government Bond is issue without discount, with maturity of 10 years, facial of 1.000€ and ten coupons of 20€ to be paid yearly. The interest rate is 2%

$$IRR \Rightarrow -1.000 + \frac{20}{1+r} + \frac{20}{(1+r)^2} + \ldots + \frac{1.020}{(1+r)^{10}} = 0 \;\Rightarrow\; IRR = 2\%$$

If the bond is sold after one year time and the interest rate remain unchanged the return is:

$$Value = \frac{20}{1,02} + \frac{20}{1,02^2} + \ldots + \frac{1.020}{1,02^9} = 1.000$$

$$\text{return} = \frac{1.000 + 20 - 1.000}{1.000} = 2\%$$

i=2%

…but… if interest rate rise to 3% or fall to 1%

$$Value = \frac{20}{1,03} + \frac{20}{1,03^2} + \ldots + \frac{1.020}{1,03^9} = 922,14$$

$$\text{return} = \frac{922,14 + 20 - 1.000}{1.000} = -5,79\%$$

i=3%

$$Value = \frac{20}{1,01} + \frac{20}{1,01^2} + \ldots + \frac{1.020}{1,01^9} = 1.085,66$$

$$\text{return} = \frac{1.085,66 + 20 - 1.000}{1.000} = 10,57\%$$

i=1%

Riesgo de tipo de interés - Reinversión

Reinvestment risk

Imagine: you have to pay 1.000.000 € in five years time, and current interest rate is 8% so you will need now:

$$\frac{1.000.000}{(1+0{,}08)^5} = 680.583{,}20$$

You buy a bond with facial of 680.583,20€, and 5 coupons at 8%. Maturity = 5 years and Yield to Maturity = 8%

	1	2	3	4	5
- 680.583,20	54.446,66	54.446,66	54.446,66	54.446,66	735.029,85

Total value of coupons and principal = **952.816,48€**

So we reinvest at 8% the coupons:

	$\cdot(1{,}08)^4$	$\cdot(1{,}08)^3$	$\cdot(1{,}08)^2$	$\cdot(1{,}08)^1$	$\cdot(1{,}08)^0$
8%	74.074,07	68.587,11	63.506,58	58.802,39	735.029,85
	$\cdot(1{,}07)^4$	$\cdot(1{,}07)^3$	$\cdot(1{,}07)^2$	$\cdot(1{,}07)^1$	$\cdot(1{,}07)^0$
7%	**71.368,46**	**66.699,49**	**62.335,98**	**58.257,92**	735.029,85

Reinvested total value = **1.000.000,00€**

Reinvested total value = **993.691,70 €**

What if interest fails to 7%?

Riesgo de tipo de interés

Sources of interest rate risk

– **Repricing risk.** The primary and most often discussed form of interest rate risk arises from timing differences in the maturity (for fixed-rate) and repricing (for floating-rate) of bank assets, liabilities, and OBS positions.

– **Yield curve risk.**

OBS = Off Balance Sheet

– **Basis risk.** It arises from imperfect correlation in the adjustment of the rates earned and paid on different instruments with otherwise similar repricing characteristics.

– **Optionality.** An additional and increasingly important source of interest rate risk arises from the options embedded in many bank assets, liabilities, and OBS portfolios. Formally, an option provides the holder the right, but not the obligation, to buy, sell, or in some manner alter the cash flow of an instrument or financial contract.

Measurement Techniques

A. Repricing schedules
   - Gap analysis (Maturity)
   - Duration

B. Simulation
   - Static and dynamic

C. Additional issues

Behavioral maturity differs from contractual maturity

Asset side of bank balance sheet. Mortgages which can be subject to prepayment, volatility resulting from demographic (death, divorce, or job transfers) and macroeconomic)

Liability side. Non-maturity deposits such as sight deposits and savings deposits.

Fixed income – The risk of default

A Risk-Free security is a fixed income asset with no credit risk, generally speaking a debt instrument issued by the State

Private debt instruments are riskier than public debt and therefore provide a higher return = Rf + spread. This spread will depend on the credit quality of each corporate

Rating agencies analyze debt issuers and issues and provide an opinion about their creditworthiness. There are short-term and long-term ratings

Promised bond yields usually go up as ratings go down.

Moody´s	S&Ps
Aaa	AAA
Aa2	AA
A2	A
Baa2	BBB
Baa3	BBB-
Ba1 and below	BB+ and below

"Investment grade"

Fixed Income –
The risk of default (cont)

Company	Moody’s Rating
Banco Santander	Baa1
Iberdrola	Baa1
BBVA	Baa2
Gas Natural	Baa2
Telefónica	Baa2
Repsol	Baa3
Bankinter	Ba1
Grifols	Ba2
Campofrío	B1
Codere	Caa3

March 2014

For the first time since 2010 Moody’s upgraded the rating of the Kingdom of Spain to Baa2 on February 21st 2014.

This upgrade in the sovereign brought along a general improvement in the ratings of most Spanish corporates.

Resumen

- **Matemática financiera: el VAN y la TIR**
- **Conceptos fundamentales:**
  - Bonos, cupones, vencimiento y plazo
  - La curva de tipos
  - Referencias de tipos de interés
- **Riesgo y bonos:**
  - Riesgo de tipo de interés:
    - Precio
    - Reinversión
  - Riesgo de crédito

¡Gracias!

A vuestra disposición para responder todas las preguntas que queráis hacer

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