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Fall 2023

Financial Markets

L3 – Interest Rate Risk

October 1st, 2025

Summary of previous classes

Lesson 1. Financial system

Efficient-market hypothesis (Fama, 1965).

- Weak form
- Semi-strong form
- Strong form

Arbitrage concept



Lesson 2. Monetary policy

Currency
Central banks reserves
Demands deposits
Saving deposits
Large time deposits and equivalent

M0

$$\frac{M2}{M0} = \text{money multiplier}$$

1928-1992

- Hyperinflation
- Bretton Woods
- Soros

16S-1992

Bank of England: 10%-15% in 1h.

M2

i

CENTRAL BANKS:

expansionary monetary policy

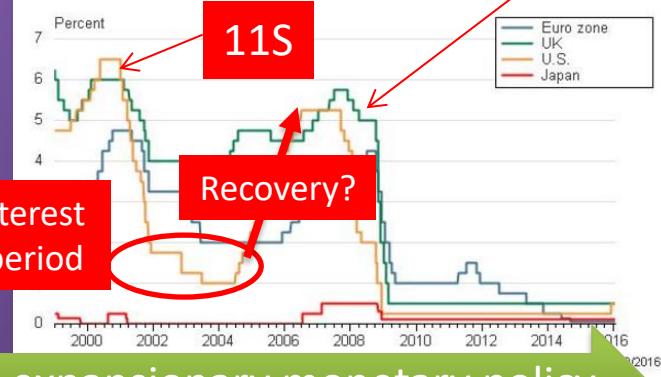
- Open market operations
- Standing facilities (interest rate)
- Maintenance of minimum reserves.

2001-2008

Central bank interest rates

11S

Recovery?



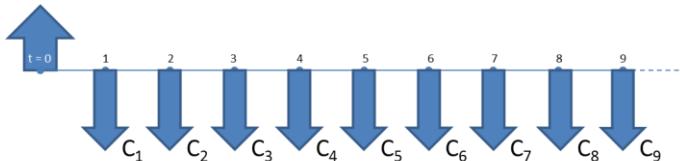
Summary of previous classes

Time value of money

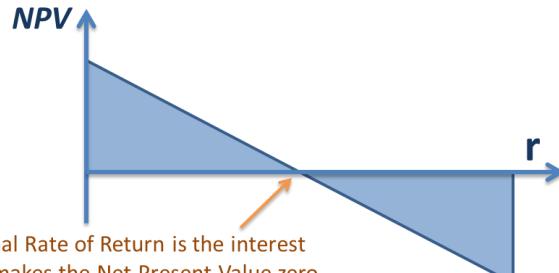
Present value – future value

$$PV = \frac{FV}{(1+r)^t}$$

Cash flow



$$NPV = \left(\frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \dots + \frac{C_9}{(1+r)^9} \right) - I$$



Given 3 you can always find the 4th

$$FV = PV(1+r)^t$$

$$PV = FV/(1+r)^t$$

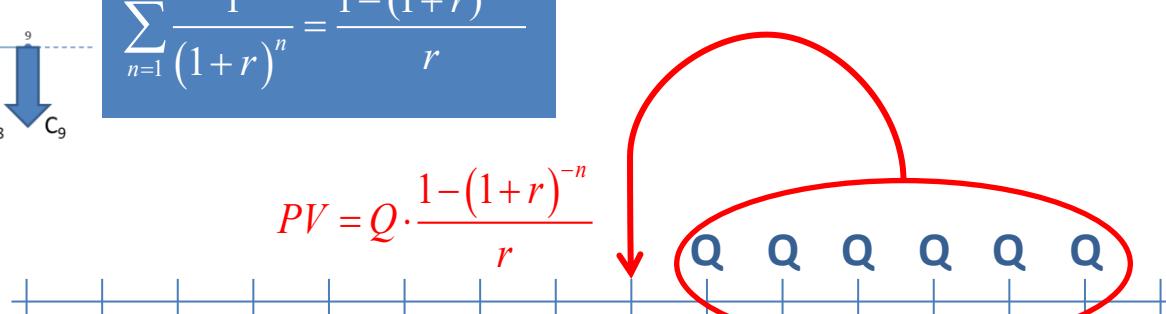
$$r = (FV/PV)^{1/t} - 1$$

$$t = \log(FV/PV)/\log(1+r)$$

Stocks & dividends
Bonds and coupons
Zero Coupon bonds
Saving
Loans
Interest rates
...

$$\sum_{n=1}^{13} \frac{1}{(1+r)^n} = \frac{1 - (1+r)^{-13}}{r}$$

$$PV = Q \cdot \frac{1 - (1+r)^{-n}}{r}$$



$$\frac{PV}{(1+r)^k} \quad \quad \quad PV(1+r)^k$$

What are we going to do?

- We will continue working with maths...
- **Interest rate risk**
 - Bond concept: maturity, price and duration
 - Interest rate risk
 - Temporal structure of interest rates
- **Review the tools of monetary policy**
- **What happened after Lehman?**
 - Systemic risk: Iceland and Greek crisis
 - Liquidity trap
 - Quantitative easing

L3. Interest rate risk

And next week... Banks:
- 2013 Nobel prize
- 2022 Nobel prize

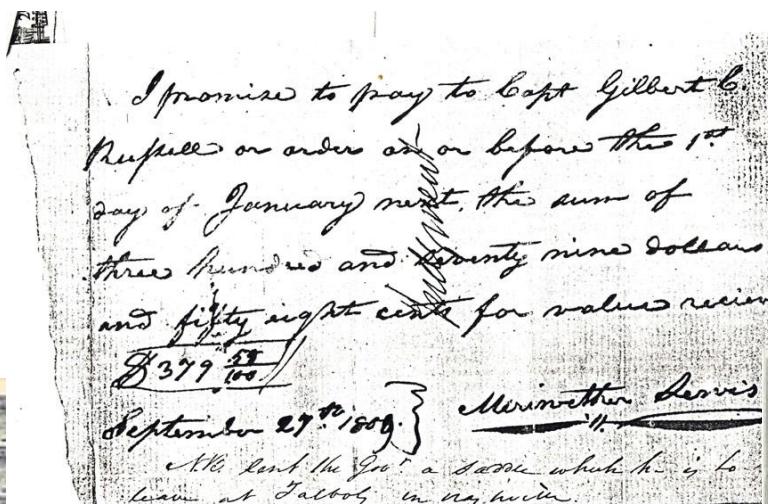


Bond concept

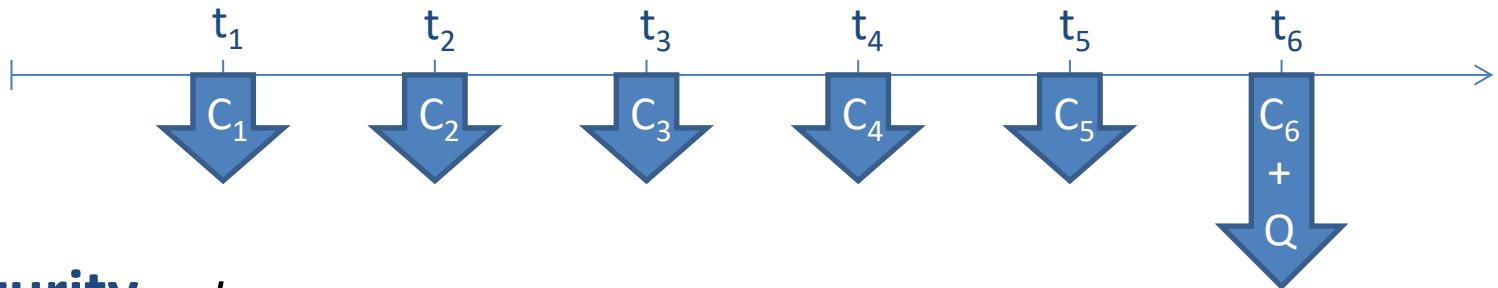
What is a Bond?

IOU note (I owe you)

- The issuer: borrower
- Investor: lender
- Face value: amount borrowed
- Maturity date: term
- Coupons: fixed at issuance



Maturity, price and duration



Maturity = t_6

$$\text{Price} = VA(C_1) + VA(C_2) + \dots + VA(C_6 + Q) = \frac{C_1}{(1+r)^{t_1}} + \frac{C_2}{(1+r)^{t_2}} + \dots + \frac{C_6 + Q}{(1+r)^{t_6}}$$

$$\text{Duration} = \frac{t_1 \cdot VA(C_1) + t_2 \cdot VA(C_2) + \dots + t_6 \cdot VA(C_6 + Q)}{VA(C_1) + VA(C_2) + \dots + VA(C_6 + Q)} = \frac{t_1 \cdot \frac{C_1}{(1+r)^{t_1}} + t_2 \cdot \frac{C_2}{(1+r)^{t_2}} + \dots + t_6 \cdot \frac{C_6 + Q}{(1+r)^{t_6}}}{\frac{C_1}{(1+r)^{t_1}} + \frac{C_2}{(1+r)^{t_2}} + \dots + \frac{C_6 + Q}{(1+r)^{t_6}}}$$

Maturity, price and duration

Calculate duration of following cash flow using a discount of 15%:

Tiempo	1	2	3
Flujo de caja	3.000,00	2.000,00	1.000,00

$$duration = \frac{1 \cdot \frac{3.000}{1,15} + 2 \cdot \frac{2.000}{1,15^2} + 3 \cdot \frac{1.000}{1,15^3}}{\frac{3.000}{1,15} + \frac{2.000}{1,15^2} + \frac{1.000}{1,15^3}} = 1,59 \text{ years}$$

Calculate duration of a zero coupon bond with maturity of 10 years and a yield to maturity of 5%. Also calculate the issue price and the discount. (Nominal=1.000€)

$$duration = maturity = 10 \text{ years}$$

$$Price = \frac{1.000}{1,05^{10}} = 613,91$$

$$Discount = \frac{1.000 - 613,91}{1.000} = 38,61\%$$

Maturity, price and duration

A Bond is issued without discount, with maturity of 3 years, facial of 1.000€ and three coupons of 100€ to be paid yearly. Calculate yield to maturity and duration (with IRR)

Time		1	2	3
Debt	- 1.000,00			1.000,00
Coupon		100,00	100,00	100,00
Cash Flow	- 1.000,00	100,00	100,00	1.100,00

$$duration = \frac{1 \cdot \frac{100}{1,1} + 2 \cdot \frac{100}{1,1^2} + 3 \cdot \frac{1.100}{1,1^3}}{\frac{100}{1,1} + \frac{100}{1,1^2} + \frac{1.100}{1,1^3}} = 2,74 \text{ years}$$

$$IRR \Rightarrow -1.000 + \frac{100}{1+r} + \frac{100}{(1+r)^2} + \frac{1.100}{(1+r)^3} = 0 \Rightarrow IRR = 10\% \Rightarrow \frac{100}{1,1} + \frac{100}{1,1^2} + \frac{1.100}{1,1^3} = 1.000$$

A Bond is issued without discount, with maturity of 10 years, facial of 1.000€ and ten coupons of 100€ to be paid yearly. Calculate yield to maturity and duration (with IRR)

Time		1	2	3	4	5	6	7	8	9	10
Debt	- 1.000,00										1.000,00
Coupon		100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00
Cash flow	- 1.000,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	1.100,00

	Total										
NPV	1.000,00	90,91	82,64	75,13	68,30	62,09	56,45	51,32	46,65	42,41	424,10
time x NPV	6.759,02	90,91	165,29	225,39	273,21	310,46	338,68	359,21	373,21	381,69	4.240,98
Duration	6,76										

Interest rate risk

A government Bond is issued without discount, with maturity of 10 years, face value of 1.000€ and ten coupons of 20€ to be paid yearly. The interest rate is 2%

$$IRR \Rightarrow -1.000 + \frac{20}{1+r} + \frac{20}{(1+r)^2} + \dots + \frac{1.020}{(1+r)^{10}} = 0 \Rightarrow IRR = 2\%$$

If the bond is sold after one year time and the interest rate remains unchanged the return is:

$$Value = \frac{20}{1,02} + \frac{20}{1,02^2} + \dots + \frac{1.020}{1,02^9} = 1.000$$

$$\text{return} = \frac{1.000 + 20 - 1.000}{1.000} = 2\%$$

i=2%

...but... if interest rate rises to 3% or falls to 1%

$$Value = \frac{20}{1,03} + \frac{20}{1,03^2} + \dots + \frac{1.020}{1,03^9} = 922,14$$

$$\text{return} = \frac{922,14 + 20 - 1.000}{1.000} = -5,79\%$$

i=3%

$$Value = \frac{20}{1,01} + \frac{20}{1,01^2} + \dots + \frac{1.020}{1,01^9} = 1.085,66$$

$$\text{return} = \frac{1.085,66 + 20 - 1.000}{1.000} = 10,57\%$$

i=1%

Interest rate risk

The interest rate risk can affect investment in two ways:

- **Changes the market value** of assets whose cash flows are independent of the interest rate .
- Changes the **cash flow's values** of those assets whose cash flows depend on the interest rate.

Liabilities with fixed rate vs.

(Market value changes and not cash flows)

If a firm's ability to sell its product is affected by the level of real interest rates, then the firm will want to minimize the exposure of its liabilities to interest rate changes – e.g. fixed rate liabilities

Higher rates: higher financial expenses and lower revenues
Lower rates: lower financial expenses and higher revenues

Modified duration. Quantifies the measurable change in the value of a security in response to a change in interest rates.

$$\text{Modified duration} = \frac{\text{Duration}}{1+r} \%$$

Liabilities with floating rates

(Cash flows change and not market value)

If changes in interest rates mainly reflect changes in inflation and if a firm's operating profits increase with inflation, then the firm will want its liabilities to be exposed to interest rate risk – e.g. floating rate liabilities

Higher rates: higher financial expenses and higher revenues
Lower rates: lower financial expenses and lower revenues

- Rate risk exposure measure
- High duration = high rate risk impact.
(Highest duration: zero coupon)
- Interest rise price falls



Term Structure of Interest Rates (TSIR)

What is the interest rate?

- EURIBOR, Central Bank Interest rate, the Yield curve, Kd?
- Who controls interests rates
- What about monetary policy?

Public finance

Government budget - (presupuestos generales del estado)

Primary market

El Tesoro anuncia los bonos y obligaciones que se subastarán el día 18 de diciembre

Detalle:
El Tesoro anuncia los Bonos y Obligaciones que se subastarán el día 18 de diciembre.

Fecha de subasta: 18 de diciembre.

Obligaciones del Estado cupón 4,60%, Vto.: 30 julio 2019

Intereses corridos: 1,84 %

Obligaciones del Estado cupón 4,40%, Vto.: 31 octubre 2023

Intereses corridos: 0,64 %

Obligaciones del Estado cupón 2,75%, Vto.: 31 octubre 2024

Intereses corridos: 0,40 %

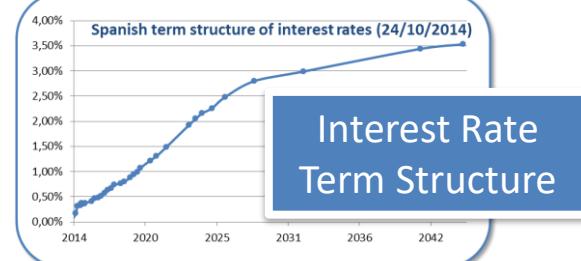


El precio que se solicite en las ofertas competitivas se formulará ex cupón.

Secondary market

EMISIÓN	Tipo	Cupón	Vencimiento	Rendimiento interno medio	Número de operaciones	Media	Máximo	Mínimo
ES0000141211	Letra del Estado	CUP-0	21/11/2014	0,06%	1	99,996	99,996	99,996
ES00000012371	Bono del Estado	2,75%	31/03/2015	0,32%	1	101,030	101,030	101,030
ES0001510160	Letra del Estado	CUP-0	16/10/2015	0,36%	4	99,653	99,653	99,651
ES00000012391	Bono del Estado	3,75%	31/10/2015	0,38%	2	103,380	103,380	103,375
ES00000012205	Bono del Estado	3,25%	30/04/2016	0,41%	7	104,241	104,245	104,210
ES00000012389	Obligación del Estado	3,80%	31/01/2017	0,52%	1	107,339	107,339	107,339
ES00000012409	Obligación del Estado	5,50%	30/04/2021	1,31%	19	125,961	126,000	125,900
ES00000012682	Obligación del Estado	2,75%	31/10/2024	2,16%	19	105,305	105,380	105,100
ES00000124AC5	Obligación del Estado	5,15%	31/10/2028	2,80%	5	126,867	127,000	126,620
ES00000012421	Obligación del Estado	5,75%	30/07/2032	2,99%	3	137,572	137,901	137,570
ES00000012157	Obligación del Estado	4,70%	30/07/2041	3,44%	1	121,880	121,880	121,880
ES00000124H4	Obligación del Estado	5,15%	31/10/2044	3,53%	4	129,738	129,770	129,700

...each day...



<http://www.civio.es/>

<http://dondevanmisimpuestos.es/resumen>

Cotizaciones sociales

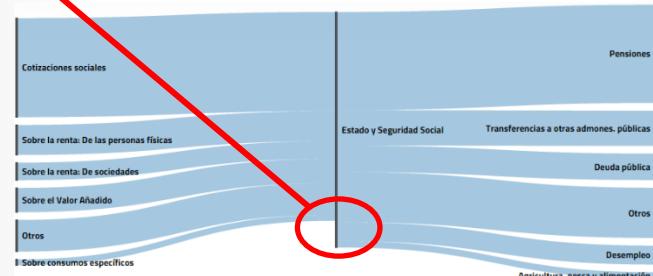
Sobre la renta

Sobre el Valor Añadido

IN

At least, with one year time:

Governments need to know, exactly what to pay, what to get and when.



<http://dondevanmisimpuestos.es/politicas>

A familias e instituciones sin fines de lucro (cap.4)

OUT

Amortización de deuda pública en moneda nacional

De deuda pública en moneda nacional



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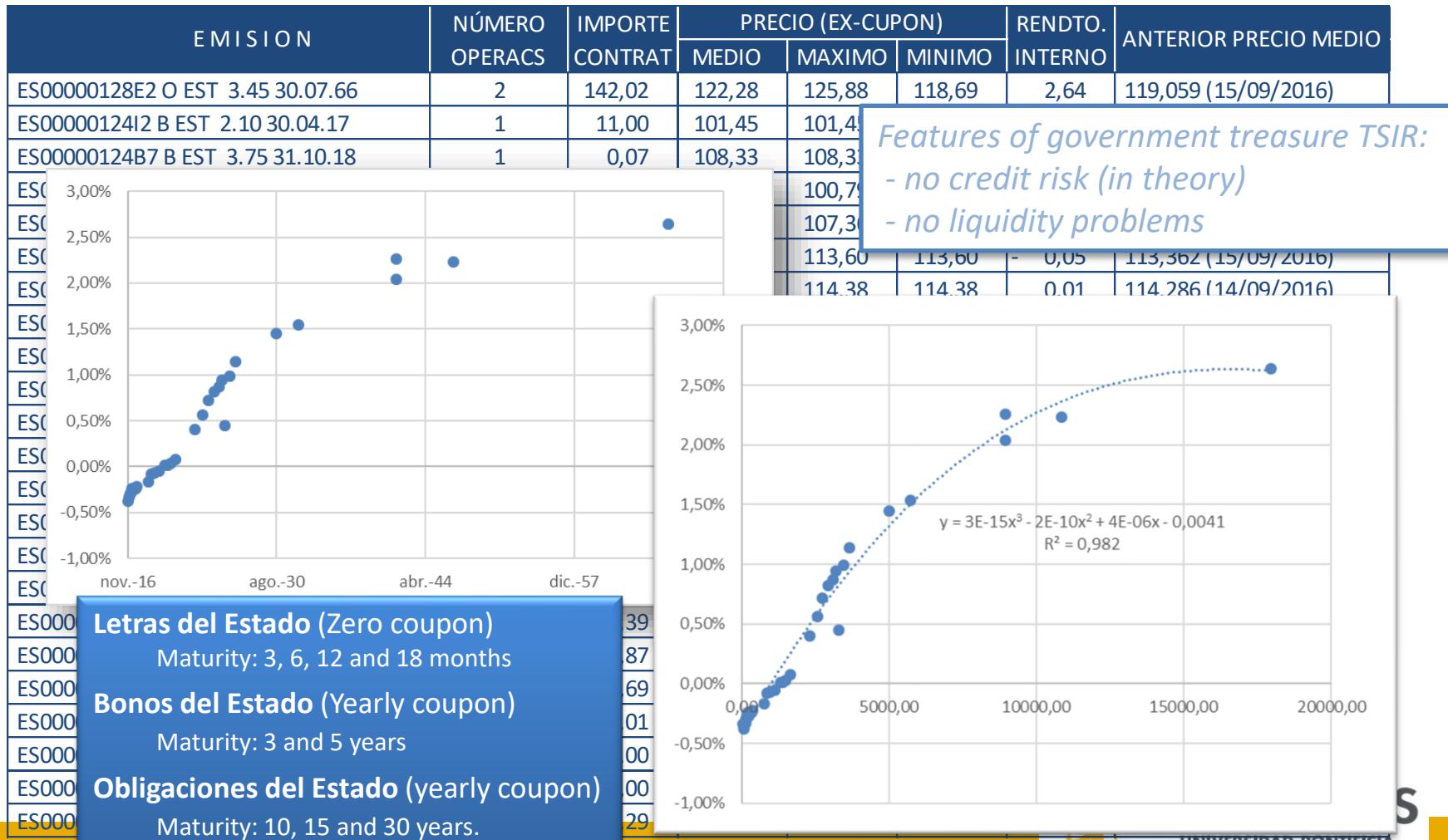
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Term Structure of Interest Rates (TSIR).

<http://www.bde.es/webbde/es/secciones/informes/banota/boletin.html>



Term Structure of Interest Rates (TSIR).



United Kingdom
Debt Management
Office

<http://www.dmo.gov.uk/>

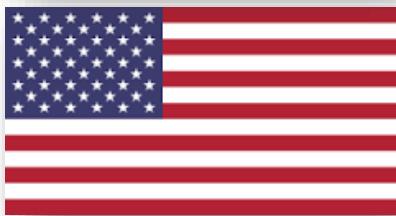
<http://www.bankofengland.co.uk/statistics/pages/yieldcurve/default.aspx>



BANK OF CANADA

<https://www.bankofcanada.ca/>

<https://www.bankofcanada.ca/rates/interest-rates/bond-yield-curves/>



U.S. DEPARTMENT OF THE TREASURY

<https://www.treasury.gov/resource-center/data-chart-center/interest-rates/>



中國銀行
BANK OF CHINA

<https://www.boc.cn/en/index.Html>

GILT MARKET							
GILT REFERENCE PRICES							
LATEST AVAILABLE PRICES							
Publication of these prices does not constitute an offer to buy or sell securities. To export the information from this report click on one of the buttons at the end of this page.							
Gilt Name	ISIN Code	Redemption Date	Close of Business Date	Clean Price	Dirty Price (E)	Accrued Interest (E)	Yield Modified Duration
Conventional							
3% Treasury Stock 2017	GB0009621148	25-Aug-2017	03-Feb-2017	104.880000	108.783234	0.022324	0.057476 0.51
3% Treasury Gilt 2017	GB0007193558	07-Sep-2017	03-Feb-2017	100.850000	103.009890	0.418890	0.01023 0.58
3% Treasury Gilt 2018	GB00091VWP0C84	07-Mar-2018	03-Feb-2017	105.300000	107.390448	2.069448	0.065553 1.04
1 1/2% Treasury Gilt 2018	GB00088KPM044	22-Jun-2018	03-Feb-2017	101.880000	101.731796	0.051796	0.067078 1.45
4 1/2% Treasury Gilt 2019	GB000839RJF94	07-Mar-2019	03-Feb-2017	109.100000	110.086503	0.886503	0.118409 1.98
2% Treasury Stock 2019	GB0009000000	25-Jun-2019	03-Feb-2017	100.000000	100.000000	0.000000	0.000000 0.00
2 1/2% Treasury Gilt 2019	GB0084YRFP41	07-Sep-2019	03-Feb-2017	108.200000	110.774598	1.574598	0.174623 2.45
4 1/2% Treasury Stock 2020	GB0008050D055	07-Mar-2020	03-Feb-2017	113.730000	115.724475	1.004475	0.270374 2.88
2% Treasury Gilt 2020	GB00BN85R198	22-Jun-2020	03-Feb-2017	105.470000	105.502873	0.062873	0.408072 3.35

Data Date: 05 Feb 2017



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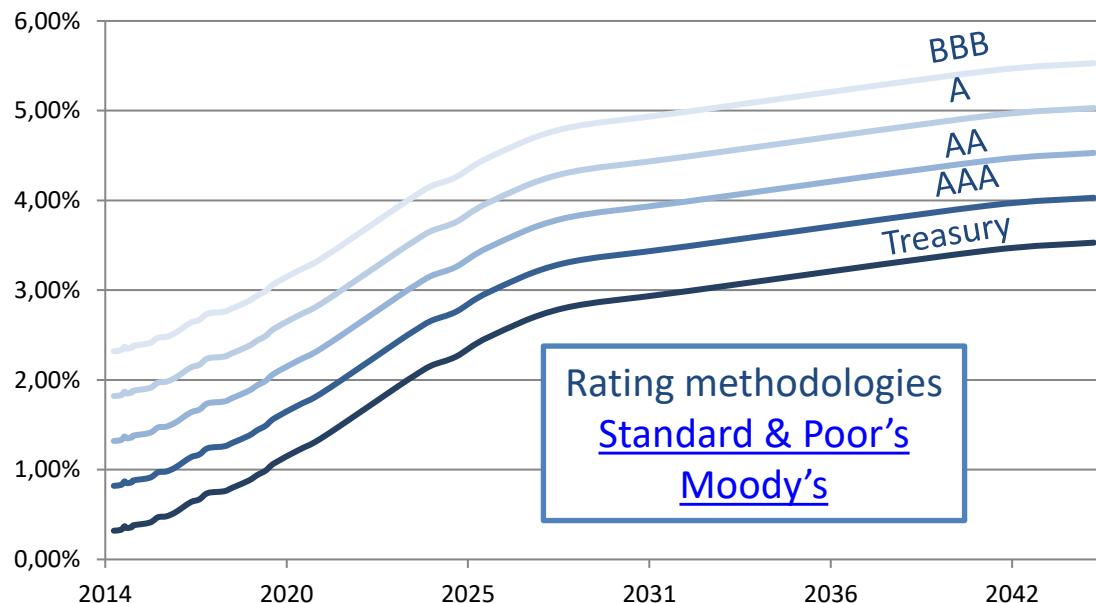
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Term Structure of Interest Rates (TSIR).

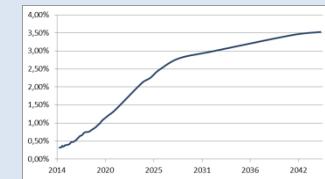
There is a different TSIR each day, and not all kind of assets fit in the same TSIR.



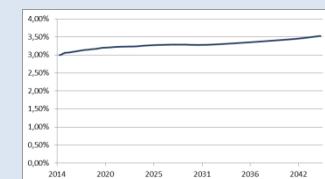
Rating methodologies
Standard & Poor's
Moody's

Types of curves

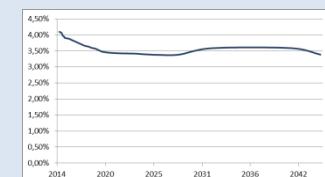
Normal curve (increasing)



Flat curve



Decreasing



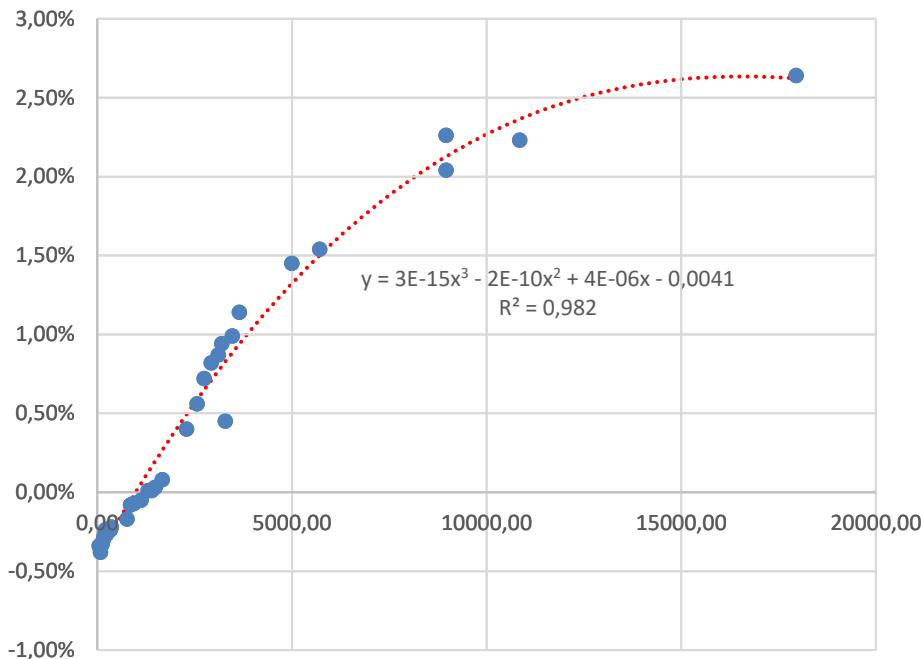
Dynamic yield – curve vs SP500

<http://stockcharts.com/freecharts/yieldcurve.php>

Term Structure of Interest Rates (TSIR).

Zero Coupon curve (I)

Using excel, first “scatter plot” and then “polynomial trend line”, we get the polynomial approximation to the TSIR.



With the formula we can get following table:

Years	IRR
1	-0,27%
2	-0,13%
3	0,00%
4	0,13%
5	0,25%
6	0,36%
7	0,48%
8	0,58%
9	0,69%
10	0,78%



Term Structure of Interest Rates (TSIR).

Zero Coupon curve (II)

A bond is equivalent to a surface of zero coupon bonds with same payments.

$$\text{IRR}_{1 \text{ year}} = -0,27\% \rightarrow \text{ZCC}_{1 \text{ year}} = -0,27\%$$

$$\text{IRR}_{2 \text{ year}} = -0,13\%$$

$$\text{IRR}_{3 \text{ year}} = 0,00\%$$

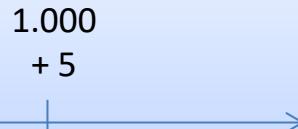


$$1.000 = \frac{-1,3}{0,9973} + \frac{998,68}{(1+\text{ZCC}_2)^2} \rightarrow \text{ZCC}_2 = -0,13\%$$



$$1.000 = \frac{0}{0,9973} + \frac{0}{(1+\text{ZCC}_2)^2} + \frac{1.000}{(1+\text{ZCC}_3)^3} \rightarrow \text{ZCC}_3 = 0,00\%$$

$\text{IRR}_3 = 5\%$



$$1.000 = \frac{50}{0,9973} + \frac{50}{(1+\text{ZCC}_2)^2} + \frac{1.050}{(1+\text{ZCC}_3)^3} \rightarrow \text{ZCC}_3$$

Year	Zero CC	IRR
1	-0,27%	-0,27%
2	-0,13%	-0,13%
3	0,00%	0,00%
4	0,13%	0,13%
5	0,25%	0,25%
6	0,37%	0,36%
7	0,48%	0,48%
8	0,59%	0,58%
9	0,70%	0,69%
10	0,80%	0,78%



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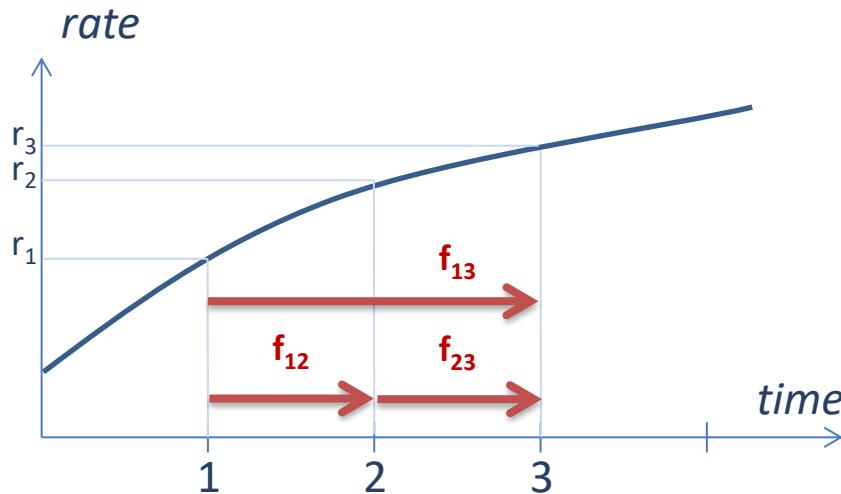
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Term Structure of Interest Rates (TSIR).

Spot rate and forward rate



Spot rate: r_1 , r_2 and r_3

Forward rate: f_{12} , f_{23} and f_{13}

$$(1 + r_2)^2 = (1 + r_1)(1 + f_{12})$$

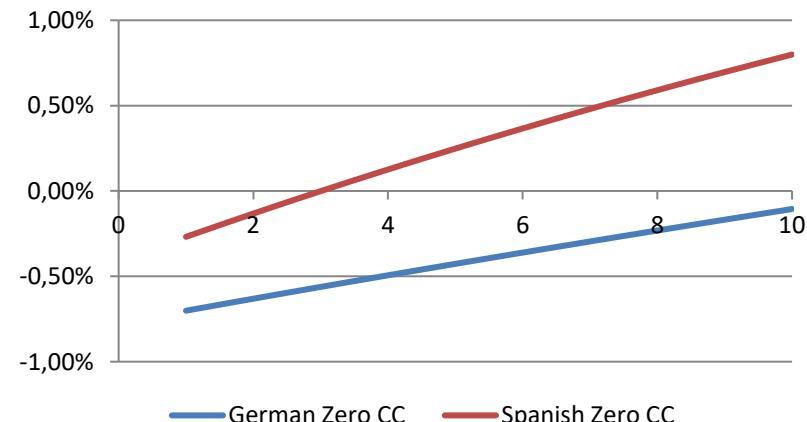
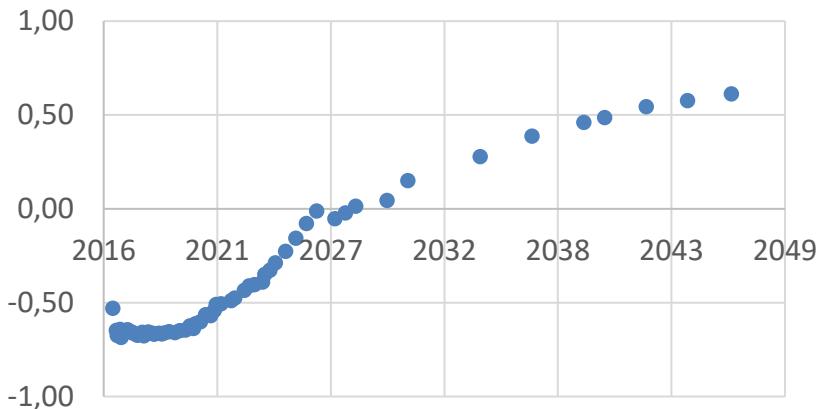
$$(1 + r_3)^3 = (1 + r_2)^2(1 + f_{23}) = (1 + r_1)(1 + f_{12})(1 + f_{23})$$

$$(1 + r_3)^3 = (1 + r_1)(1 + f_{13})^2$$

Term Structure of Interest Rates (TSIR).

German federal securities

http://www.bundesbank.de/Navigation/EN/Service/Federal_securities/Prices_and_yields/prices_and_yields.html



Year	German Zero CC	Spanish Zero CC	Difference
1	-0,70%	-0,27%	0,43%
2	-0,63%	-0,13%	0,50%
3	-0,56%	0,00%	0,56%
4	-0,49%	0,13%	0,62%
5	-0,43%	0,25%	0,68%
6	-0,36%	0,37%	0,73%
7	-0,30%	0,48%	0,78%
8	-0,23%	0,59%	0,82%
9	-0,17%	0,70%	0,86%
10	-0,11%	0,80%	0,90%

Tools of monetary policy

- **Remembering lesson 2: What are the three tools ECB can use?**
 - standing facilities,
 - minimum reserve requirements for credit institutions,
 - open market operations.

Is it really useful now?

What is the aim of the ECB?

Let see the evolution of the financial system...

Open market operations.	Transaction type		Maturity	Frecuency	Procedure
	Liquidity injection	liquidity absorption			
Main refinancing operations	Temporary operations		2 weeks	Weekly	Standard tenders
Longer-term refinancing operations	Temporary operations		3 months	Monthly	Standard tenders
Fine-tuning operations	Temporary operations	Temporary operations	No standarized	No regular	Fast tenders
	Currency Swaps	Currency Swaps Collection of deposits			Bilateral procedures
Structural operations	Securities purchase	Sale of securities		No regular	Bilaterals procedures
	Temporary operations	Issuance of debt certificates	Standarized / no st	Regular and n	Standard tenders
	Securities purchase	Sale of securities		No regular	Bilaterals procedures



Evolution of the financial system

We talked in previous classes about:

- 1921-1924 German hyperinflation
- 1944-1971 Bretton Woods. Gold standard (fixed exchange rates). Stability
- 1979-1998 European Monetary System. 1992 Soros and the pound.
- 2001 The Federal Reserve's Response to the Sept. 11 Attacks
- 2001-2008 Low interest rate period with the “recovery” from 2006.
- **Sept 2008 – Lehman Brothers Bankruptcy**

...and then?

Systemic risk

Zero interest rate policy

Iceland crisis
Greek crisis

Liquidity trap
Quantitative easing



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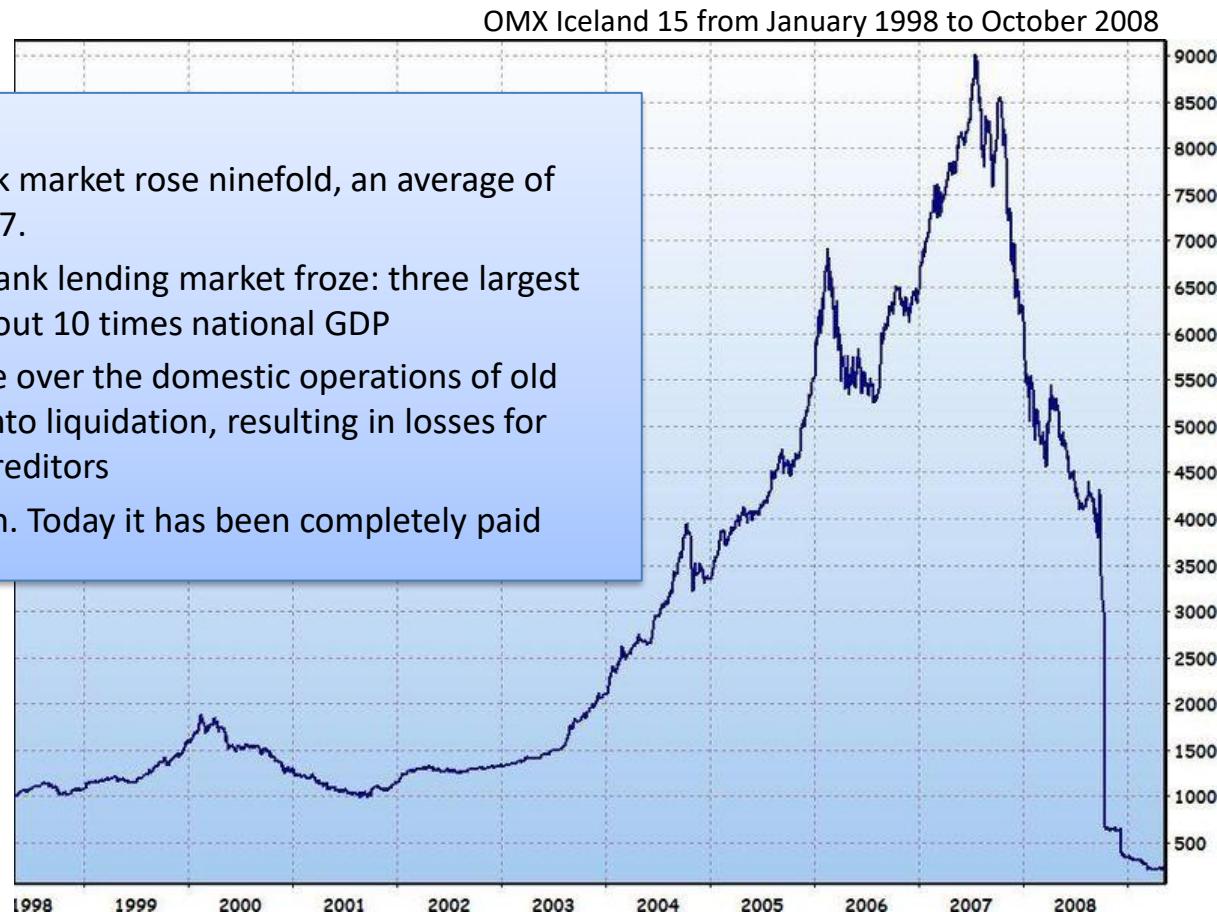
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Systemic risk: Iceland crisis

- 2000 Banks were privatized.
- Banks borrowed heavily. Ej: stock market rose ninefold, an average of 44% each year from 2001 to 2007.
- After Lehman bankruptcy interbank lending market froze: three largest Banks had liabilities equaling about 10 times national GDP
- New banks were founded to take over the domestic operations of old banks. The old banks were put into liquidation, resulting in losses for their shareholders and foreign creditors
- IMF agreed to lend €1.875billion. Today it has been completely paid



Systemic risk: Greek crisis

- Officially started in 2009
- Causes:
 - Global recession
 - Structural weaknesses of the Greek economy
 - Government debt levels and deficits were undercounted by the Greek government.
- Crisis of confidence & CDS

As previously happened with mortgages, different kind of debts were packaged and sold in **CDOs (Collateralized Debt Obligation)** and the credit risk was assured buying a **CDS (Credit Default Swaps)**

Greece Paid Goldman \$300 Million To Help It Hide Its Ballooning Debts [Business insider Feb 2010](#)



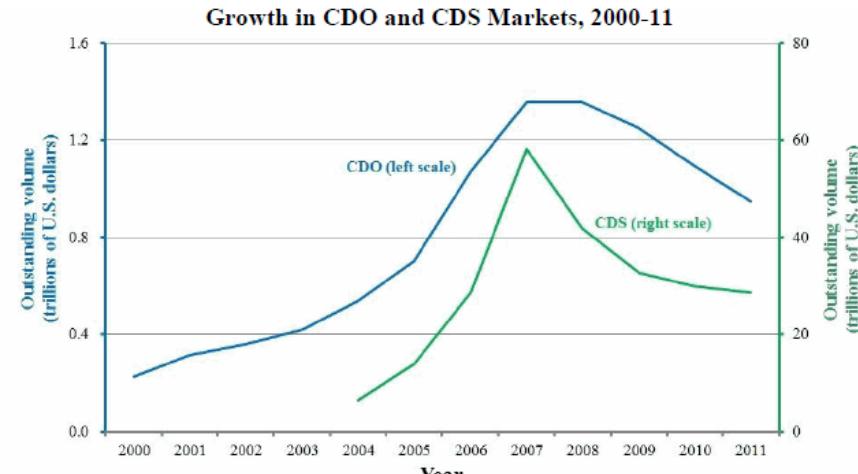
Henry Blodget



Feb. 14, 2010, 9:27 AM

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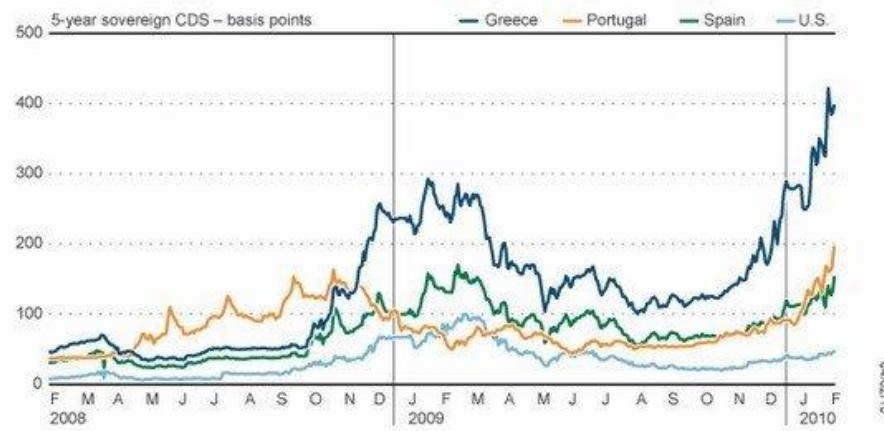


Note: CDO is collateralized debt obligation; CDS is credit default swap. CDS data start in 2004.

Source: For CDO, Securities Industry and Financial Markets Association; for CDS, Bank for International Settlements.

Global sovereign credit default swaps

The cost of insuring sovereign debt against default is rising, particularly in Europe



Source: Thomson Reuters Datastream

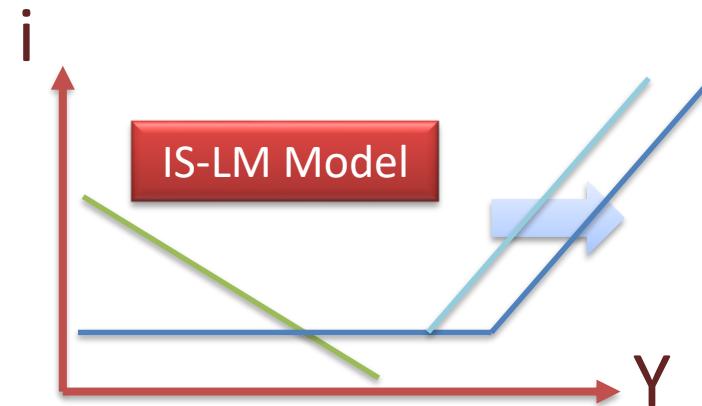
Reuters graphic/Stephen Culp

REUTERS

Zero interest rates policy: Liquidity trap

- Low interest rates and high savings rates
- Investors choose to avoid bonds and keep their funds in savings, because of the prevailing belief that interest rates will soon rise.
- **Traditional monetary policy doesn't work**

If we think that interest rates are going to rise
¿what would happen if we buy public debt?



We have two problems:

- Traditional monetary policy doesn't work
- People don't want to buy public debt

¿Solution? quantitative easing



Quantitative easing

Expanded Asset Purchase Programme (APP) [link](#)

- third covered bond purchase programme (CBPP3)
- asset-backed securities purchase programme (ABSPP)
- public sector purchase programme (PSPP)
- corporate sector purchase programme (CSPP)

How does monetary policy work?

Monthly purchases in public and private sector securities amount to €80 billion on average

Eurosystem holdings under the expanded asset purchase programme

What are the risks?

Changes of holdings (last two months)	ABSPP	CBPP3	CSPP	PSPP	APP
Holdings* July 2016	20,368	186,634	13,214	944,852	1,165,068
Monthly net purchases	-226	3,504	6,707	50,513	60,498
Quarter-end amortisation adjustment	0	0	0	0	0
Holdings* August 2016	20,142	190,139	19,921	995,364	1,225,566

*At amortized cost, in euro million, at month end.

Figures may not add up due to rounding. Figures are preliminary and may be subject to revision.



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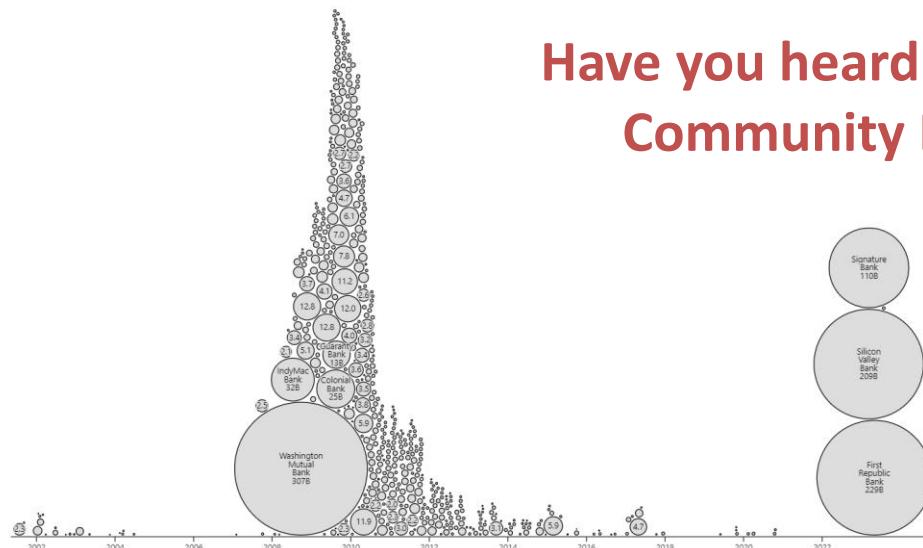
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Silicon Valley Bank Crisis

- High exposure to bonds with a maturity of over 10 years.
- The FED raises interest rates by more than 400 basis points in less than a year.
- After the pandemic, technological clients withdraw deposits to cover layoffs and operational expenses.
- Significant losses occur due to the depreciation of long-term bonds held on the balance sheet.
- The FED intervenes to prevent contagion and stabilize the financial system.



Have you heard of New York
Community Bancorp?

<https://observablehq.com/@mbostock/bank-failures>

Past, present and Future

1929 financial crisis

- The 1929 crisis was a financial crisis. As the U.S. government did not intervene, it turned into the Great Depression, lasting for years.

2008 financial crisis

- The 2008 crisis was also a financial crisis. However, immediate government intervention prevented it from becoming an economic depression.
- Ben Bernanke, then Chairman of the FED, had studied the 1929 crisis and implemented swift measures, which earned him the Nobel Prize in Economics.

Post-SVB crisis management

- Bank crises after SVB in March 2023 occur in real-time, and the FED manages these crises in real-time, often without public awareness.

What we have done until now?

Lesson 1.

Introduction

Spanish financial sector

Function of Financial Markets and financial intermediaries

Financial Market instruments

Understanding financial markets through the Financial Crisis
(Subprime)

Evolution of the financial system

An overview of Financial Risks

Lesson 2.

EMU and monetary policy

Central Banks: origins, structures and functions

The European Central Bank (ECB) and the Monetary policy

The creation of money: What is money?

Supply and demand: monetary base

Monetary policy

Lesson 3.

Interest rate risk and money markets

Concepts and classes

Understanding interest rates

The yield curve (risk and term structure)

Interbank market

Commercial paper and repos

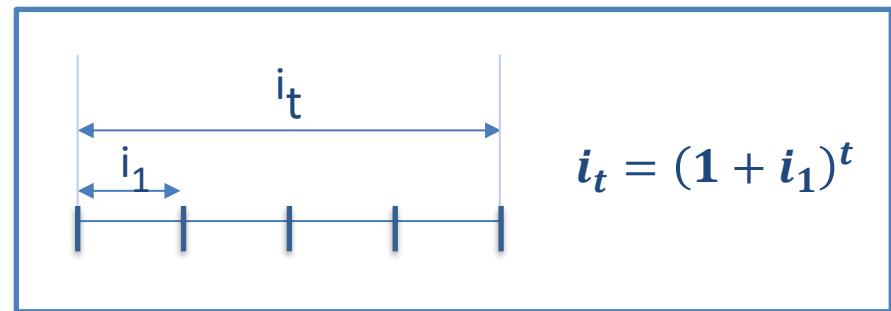
Interest rates



Performance Measurement

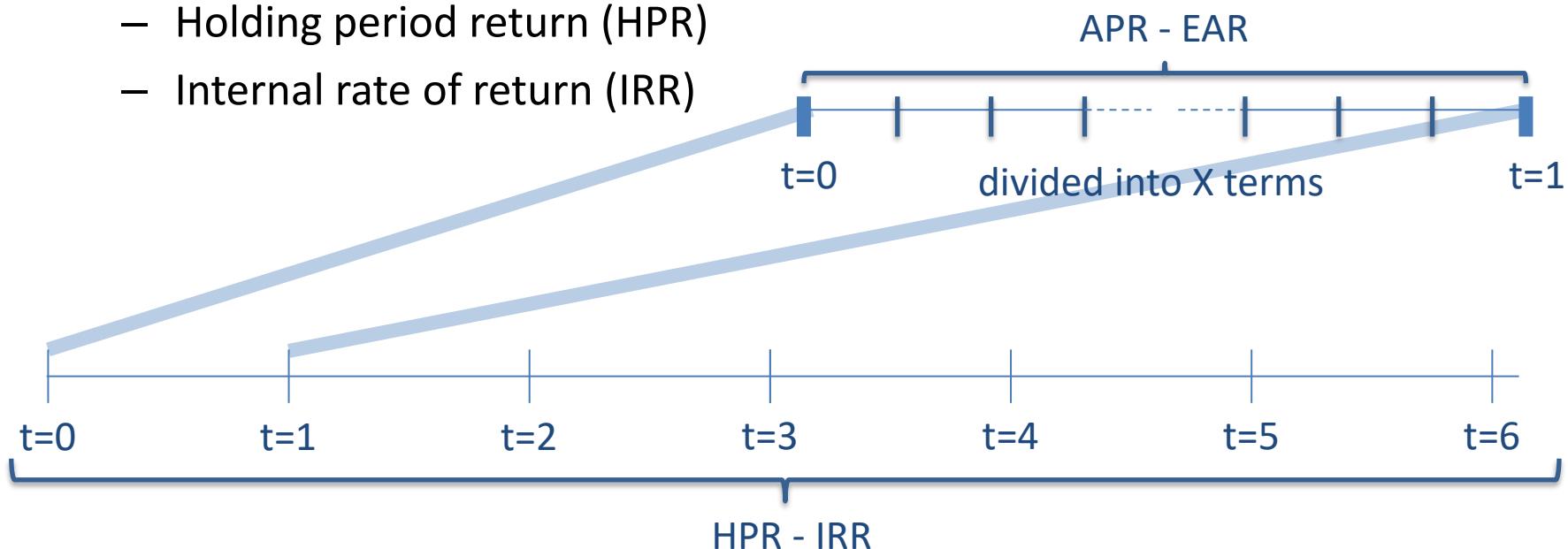
- **Compounding**

- Annual percentage rate (APR)
- Effective annual rate (EAR)



- **Multi-period returns**

- Holding period return (HPR)
- Internal rate of return (IRR)



Performance Measurement

APR (Annual Percentage Rate)

- Lenders are required by law to report the annual percentage rate (APR)
 $APR = (\text{rate per period})(\# \text{ of periods per year})$
- For example, a loan that charges 1% per month:
 $APR = 1\%(12) = 12\%$
- Is 12% per year the “real” cost?

APR vs. EAR (Effective Annual Rate)

- Effective annual rate (EAR).
If interest is compounded m times a year:

$$EAR = \left(1 + \frac{APR}{m}\right)^m - 1$$

- Which loan is cheaper?
 - 12%, compounded annually
 - 12%, compounded monthly

Performance Measurement

Continuous Compounding

- Consider increasingly frequent compounding: annually, quarterly, daily, every second,...
- When compounding happens “all the time,” it is called continuous compounding
- What happens to the EAR?

$$\text{EAR} = \exp(\text{APR}) - 1 = e^{\text{APR}} - 1$$

- Future value and present value

$$FV = PV e^{rt} \quad PV = FV e^{-rt}$$

Recap:

$$\text{EAR} = \left(1 + \text{APR}/m\right)^m - 1$$

$$\text{APR} = [(1 + \text{EAR})^{1/m} - 1]m$$

$$\text{EAR} = e^{\text{APR}} - 1$$

$$\text{APR} = \ln(1 + \text{EAR})$$

$$\frac{1}{(1+i)^t} \approx e^{-t \cdot i}$$

How do you make a loan seem cheaper?

<https://www.contante.es/terminos/>



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Performance Measurement

Holding Period Return (HPR)

The holding period return (HPR) is the rate (return) that your initial investment earned in order to generate the final value of the investment

The annual holding period return (ann. HPR) is the corresponding rate per year (annual return)

- Example
 - At time 0, you buy an investment for V_0 (PV)
 - You re-invest all intermediate cash flows until date t
 - At time t , you sell the investment *and the re-invested cash-flows* for a total price of V_t (FV)
- The annual holding period return (ann. HPR) is the solution to:

$$V_0(1 + \text{ann. HPR})^t = V_t$$

$$\text{ann. HPR} = \left(\frac{V_t}{V_0} \right)^{1/t} - 1$$

Performance Measurement

Holding Period Return (HPR) - Stock

Holding period return:

$$HPR = \frac{\text{ending price} + \text{ending cash dividend}}{\text{beginning price}} - 1$$

Annual holding period return for a holding period of t years:

$$\begin{aligned} \text{ann. HPR} &= (1 + HPR)^{1/t} - 1 \\ &= \left(\frac{\text{ending price} + \text{ending cash dividend}}{\text{beginning price}} \right)^{1/t} - 1 \end{aligned}$$

You bought Coca-Cola shares for \$42.39 and sold them 6 months later for \$44.30. Suppose there was no dividend payment in these 6 months. What are the HPR and ann. HPR?

$$HPR = 44.30/42.39 - 1 = 1.0451 = 4.51\% \text{ per 6 months}$$

Annual HPR?

$$t = 0.5$$

$$r = (44.3/42.39)^2 - 1 = 9.21\%$$

You bought KO shares for \$39.63 two years ago and sold them for \$42.37. Assume that the only dividend of \$1.12 is paid at the end of year 2. What are the HPR and ann. HPR?

$$HPR = (42.37 + 1.12) / 39.63 - 1 = 9.74\% \text{ for the entire 2 year period.}$$

$$\text{annual HPR} = ((42.37 + 1.12) / 39.63)^{(1/2)} - 1 = 4.76\%$$



Performance Measurement

Holding Period Return (HPR) – Zero Coupon Bond

$F = \$1000, PV = \$435, t = 10 \text{ years}$

$$r = (1000/435)(1/10)-1 = 0.0868 = 8.68\%$$

This r is the YTM but also the ann. HPR if you hold the bond until maturity

What is the ann. HPR if you sell early?

After 1 year for \$472.758?

$$\text{HPR} = 472.758/435 -1 = 8.68\%.$$

What is the YTM for those who buy the now 9 year bond from you?

$$\text{YTM} = (1000/472.758)^{(1/9)} -1 = 8.68\%.$$

After 1 year for \$480?

$$\text{HPR} = 480/435 -1 = 10.34\% \text{ HPR} > \text{YTM} = 8.68\%.$$

$$\text{YTM} = (1000/480)^{(1/9)} -1 = 8.50\%.$$

After 1 year for \$460?

$$\text{HPR} = 460/435 -1 = 5.75\% = \text{HPR} < \text{YTM}$$

$$\text{YTM} = (1000/460)^{(1/9)} -1 = 9.01\%.$$

Performance Measurement

IRR (Internal Rate of Return)

A third way of calculating an average return on a multiple-period investment is the internal rate of return.

- Average return
- Return if one can re-invest cash-flows at this rate
- Rate that makes **initial price = present value of future cash flows**

$$P_0 = PV = \sum_{t=1}^{\infty} \frac{C_t}{(1 + IRR)^t}$$

IRR of a Zero Coupon Bond

$F = \$1000$, $PV = \$435$, $t = 10$ years

$$P_0 = PV = \sum_{t=1}^{\infty} \frac{C_t}{(1 + IRR)^t}$$

$$PV = \frac{F}{(1 + IRR)^t} \Rightarrow IRR = \left(\frac{F}{PV} \right)^{1/t} - 1$$

$$IRR = (1000/435)^{(1/10)} - 1 = 0.0868 = 8.68\%$$

IRR=YTM=ann. HPR



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Performance Measurement

IRR (Internal Rate of Return): dividends

Coca Cola shares

- Bought 1 share 2 years ago for \$39.63
- Earned dividend of \$1.12 at end of each year
- Sell the share for \$44.30

What is the IRR?

<u>0</u>	<u>1</u>	<u>2</u>
	1.12	44.30+1.12

$$P = 39.63 = 1.12/(1+IRR) + (1.12+44.30)/(1+IRR)^2 = 0$$

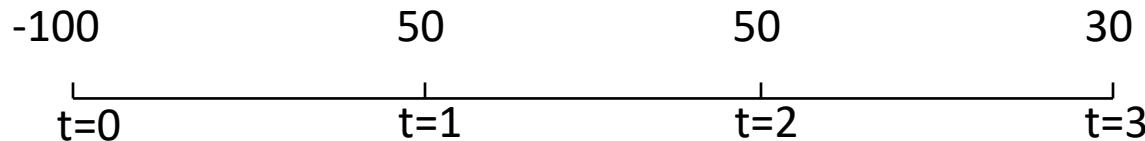
$$IRR = 8.4785\%$$

What is the ann. HPR if you can reinvest the dividend at the IRR?

Performance Measurement

IRR (Internal Rate of Return): project valuation

- Pfizer wants to compute IRR on a potential project
- Business plan projects following cash-flows:



- Solve:

$$-100 + \frac{50}{(1 + IRR)} + \frac{50}{(1 + IRR)^2} + \frac{30}{(1 + IRR)^3} = 0$$

- Answer: IRR=15.655%

$$PV = \frac{FV}{(1 + r)^t}$$

CONCLUSION

There are a number of related (in some cases identical) ways of thinking about the return on (or the cost of) an investment:

- EAR
- ann. HPR
- IRR

Thanks