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Fall 2025

Financial Markets

Practical review

September 9st, 2026

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Content

- Finance Axioms
- No Arbitrage Condition
- Buying and selling on margin
- Selling short
- Time value of money
 - Future Value, Present Value, Time and Yield
 - Multiple Cash Flows
 - Perpetuities and Annuities
 - Continuous Compounding



Finance axioms

- Finance is based on few simple axioms
- Three axioms about investor behavior
 - Investors prefer **more to less**
 - Investors are **risk averse**
 - Investors prefer payoffs **sooner rather than later**
- One about markets:
 - No arbitrage condition (“no free lunch”): Financial markets are highly competitive

Axiom 4: No Arbitrage Condition

- Financial markets are highly competitive $\Rightarrow$ all riskless profit opportunities are traded away.
- Suppose you observe the following prices:

Item	Price
Big Mac	\$4
Soda	\$2
Fries	\$3
Toy	\$2
Happy Meal	\$10

- If fast food market worked like financial markets, what is the riskless arbitrage you could undertake?



No Arbitrage Condition

- Financial markets are highly competitive $\Rightarrow$ all riskless profit opportunities are traded away.
- Suppose you observe the following prices:

Item	Price
Big Mac	\$4
Soda	\$2
Fries	\$3
Toy	\$2
Happy Meal	\$10

- If fast food market worked like financial markets, what is the riskless arbitrage you could undertake?
- Answer: Buy a Happy Meal, sell its constituent parts

No Arbitrage Condition-Example 2

- Suppose analysts have uncovered the following pattern in the data:

Stock	Rain	Shine
Parasol Co	-10%	20%
Umbrella Co	20%	-10%

- You can borrow and lend at 4% interest rate
- How can you make a sure profit with zero initial cash outlay?
 - A. Buy Parasol, sell Umbrella
 - B. Buy Parasol, buy Umbrella
 - C. Buy Parasol, buy Umbrella and borrow at 4%

No Arbitrage Condition-Example 2

- Let's write out the payouts in dollar terms:

Stock	Today	Rain	Shine
Parasol Co			
Umbrella Co			
Borrow			
Profit/loss			

- Should we expect this to last?
- What will happen if the interest rate is 5%?

No Arbitrage Condition-Example 2

- Let's write out the payouts in dollar terms:

Stock	Today	Rain	Shine
Parasol Co	-100	90	120
Umbrella Co	-100	120	90
Borrow	200	$-200 \times 1.04 = -208$	$-200 \times 1.04 = -208$
Profit/loss	Total CF=0	$210 - 208 = 2$	$210 - 208 = 2$

- Should we expect this to last? No. In an efficient market, all the gains will be arbitrated away
- What will happen if the interest rate is 5%? At 5%, the arbitrageurs will only break even

Buying and Selling on Margin

- Two types of margin requirement
 - Initial margin: represents the minimum amount an investor must put up to purchase a stock
 - Maintenance margin: represents the minimum amount required to be maintained in the margin account to maintain an open position
- In the US, regulations govern the margin requirement.
 - For a long position, the Federal Reserve Regulation T initial margin requirement is 50% and the maintenance margin requirement is 25%
 - For a short position, the initial margin requirement is 150% and maintenance margin is 130%

Buying on Margin

- Suppose you want to invest in GameStop by borrowing money from your broker (“on margin”).
- Suppose it costs \$50 today and you have \$5,000 to invest. You can borrow at 1% rate.
- Suppose further that the price can end up at \$75, \$50 and \$25.
- Let us first look at the returns from investing \$5,000 without margin

Position	Cost Today	Value Next Year	Net Return
100 shares	\$5,000	\$7,500	$7,500/5000-1=50\%$
100 shares	\$5,000	\$5,000	0%
100 shares	\$5,000	\$2,500	-50%

Buying on Margin

- Now suppose in addition to investing your own money, you borrow another \$5,000. You can now invest \$10,000
- Next year, you need to pay back $\$5,000 \times 1.01 = \$5,050$
- What are your investment returns if you invest entire \$10,000 in GameStop?

Position	Cost Today	Value Next Year	Net Return
200 shares	\$10,000	\$15,000- \$5,050=\$9,950	$9,950/5000-1=99\%$
200 shares	\$10,000	??	??
200 shares	\$10,000	??	??

Buying on Margin

- Now suppose in addition to investing your own money, you borrow another \$5,000. You can now invest \$10,000
- Next year, you need to pay back $\$5,000 \times 1.01 = \$5,050$
- What are your investment returns if you invest entire \$10,000 in GameStop?
- Leverage doubles the gain and more than doubles the loss

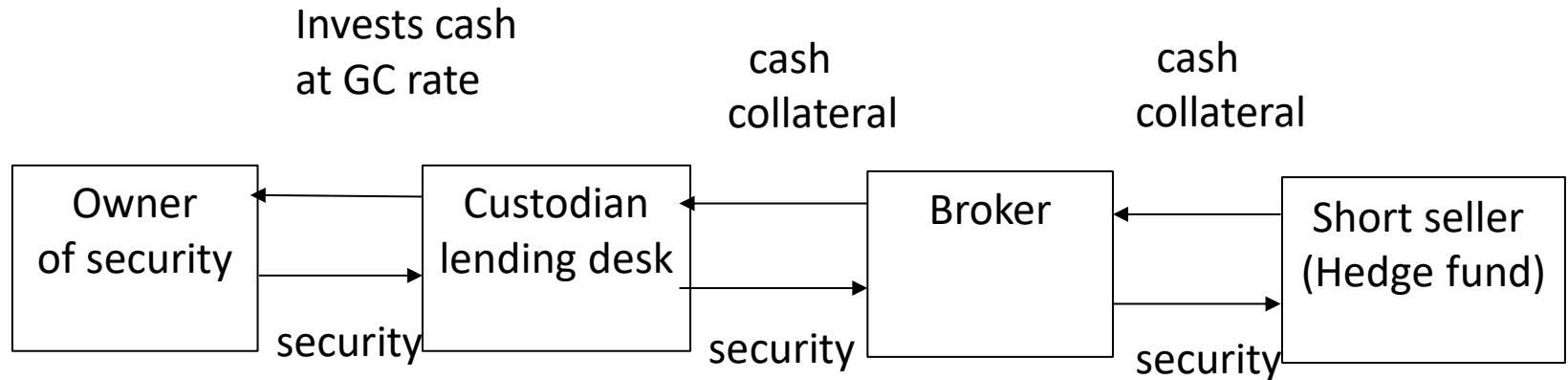
Position	Cost Today	Value Next Year	Net Return
200 shares	\$10,000	$\$15,000 - \$5,050 = \$9,950$	$9,950/5000 - 1 = 99\%$
200 shares	\$10,000	$\$10,000 - \$5,050 = \$4,950$	$4,950/5000 - 1 = -1\%$
200 shares	\$10,000	$\$5,000 - \$5,050 = -\$50$	$-50/5000 - 1 = -101\%$

Short Sale

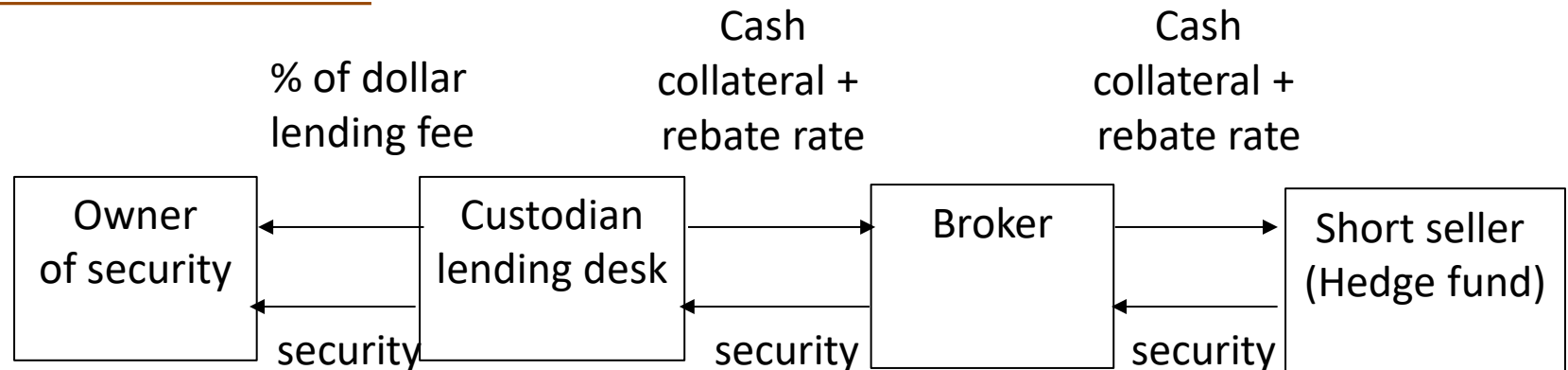
- Short sales-to sell something you don't own
 - Selling shares of a firm not owned by borrowing security, and later replacing it (cover it)
 - A profit is made if the short position is covered at a price lower than the one at which it was established
 - Bearish investment or as a hedge
- Requirements
 - Short sale proceeds must remain with the broker
 - The investor is also required to deposit collateral (to post margin) as a guarantee against default. The margin requirement is 50%
 - Short selling is subject to an uptick rule

Short Sale

At the time of Short



When short is covered



Short Selling

- Suppose now you think GameStop is overvalued.
- You decide to sell 100 shares at \$50.
- Your broker charges \$1 fee for borrowing the shares
- What would your profit/loss be if the stock goes to \$75, \$50, \$25?

Position	Value Today	Cost Next Year	Profit/(Loss)
100 shares	\$5,000	\$7,500	$\$5,000 - \$7,500 - \$1 = (\$2,501)$
100 shares	\$5,000	??	??
100 shares	\$5,000	??	??

Short Selling

- Suppose now you think GameStop is overvalued.
- You decide to sell 100 shares at \$50.
- Your broker charges \$1 fee for borrowing the shares
- What would your profit/loss be if the stock goes to \$75, \$50, \$25?
- Gain is capped at the short proceeds; the loss is not capped

Position	Value Today	Cost Next Year	Profit/(Loss)
100 shares	\$5,000	\$7,500	$\$5,000 - \$7,500 - \$1 = (\$2,501)$
100 shares	\$5,000	\$5,000	$\$5,000 - \$5,000 - \$1 = (\$1)$
100 shares	\$5,000	\$2,500	$\$5,000 - \$2,500 - \$1 = \$2,499$

Volkswagen Short-Squeeze

theguardian

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Business > Volkswagen (VW)

VW goes from the Beetle to the world's most valuable company

VW becomes world's most valuable firm

David Gow, European business editor
The Guardian, Wednesday 29 October 2008

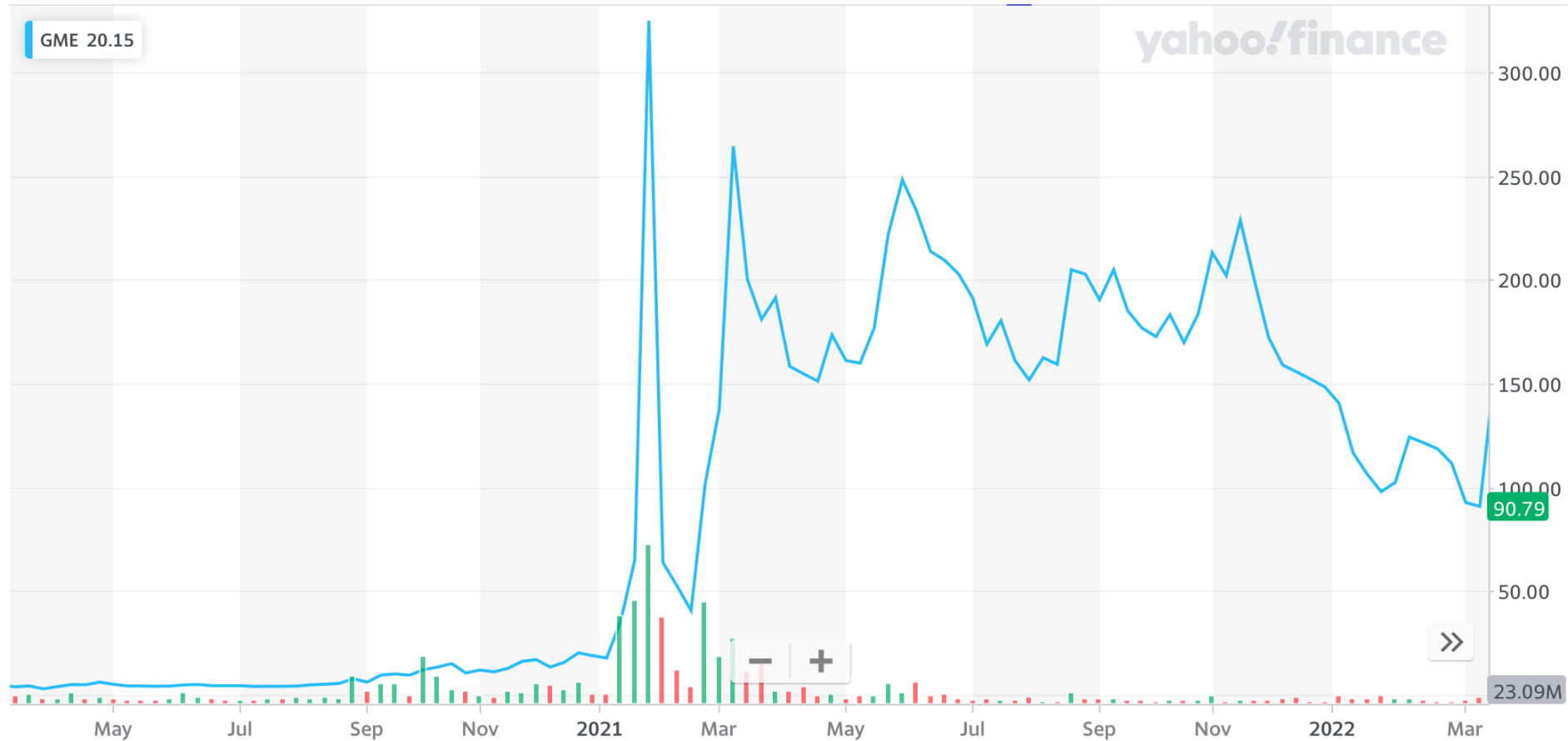


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Volkswagen Short-Squeeze



Game Stop



Buying on Margin-Example

- An investor buys 10,000 shares at \$10 each in the margin account. The initial margin is 50% and the maintenance margin is 25%. Calculate the threshold price that would trigger a margin call.
- What if the stock price decreases to \$6? How much should the hedge fund deposit for the maintenance margin requirement to be met?

Buying on Margin-Example

- An investor buys 10,000 shares at \$10 in a margin account. Initial margin 50%, maintenance margin 25%. Find the price that triggers a margin call.
- What if the price falls to \$6? How much must be deposited?
- Equity = $10,000P - 50,000$; call when equity/value < 25%
- $(10,000P - 50,000)/10,000P = 0.25 \rightarrow P = \6.67
- At \$6: equity \$10,000 vs required \$15,000 $\rightarrow$ deposit \$5,000



Selling Short-Example

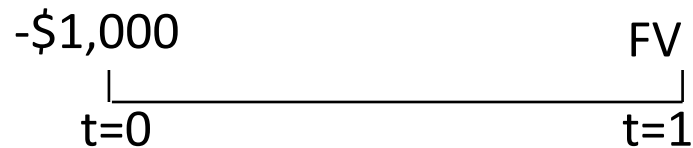
- A hedge fund deposits \$50,000 on a margin account. Suppose that the hedge fund sells short 10,000 shares at \$10. Its broker applies a 150% initial margin and a 130% maintenance margin requirement
 - Calculate the threshold stock price that triggers a margin call and calculate the new hedge fund account.
 - What if the stock price increases to \$12? How much should be deposited for the maintenance margin requirement to be met?

Selling Short-Example

- A hedge fund deposits \$50,000 and shorts 10,000 shares at \$10. Initial margin 150%, maintenance 130%.
 - Account = \$150,000; position value = 10,000P
 - Call when $150,000/10,000P < 1.30 \rightarrow P = \11.54
- At \$12: required \$156,000 $\rightarrow$ deposit \$6,000

Future Value

- What is the future value of \$1,000 invested for one year at an annual rate of 5%



$$FV = PV \times (1 + r)^T = 1,000 \times (1.05)^1 = \$1050$$

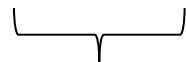
Future Value

- What is the value of \$1,000 invested for two years at an annual rate of 5%?

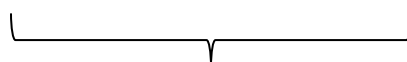
$$FV = PV \times (1 + r)^T = 1,000 \times (1.05)^2 = \$1102.5$$

- The final payment can be decomposed as:

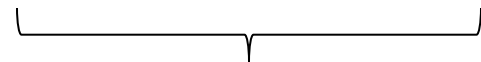
$$1102.5 = \$1000 + 2 \times 0.05 \times \$1000 + 0.05 \times 0.05 \times \$1000$$



Principal



Simple Interest



Interest on Interest



Future Value

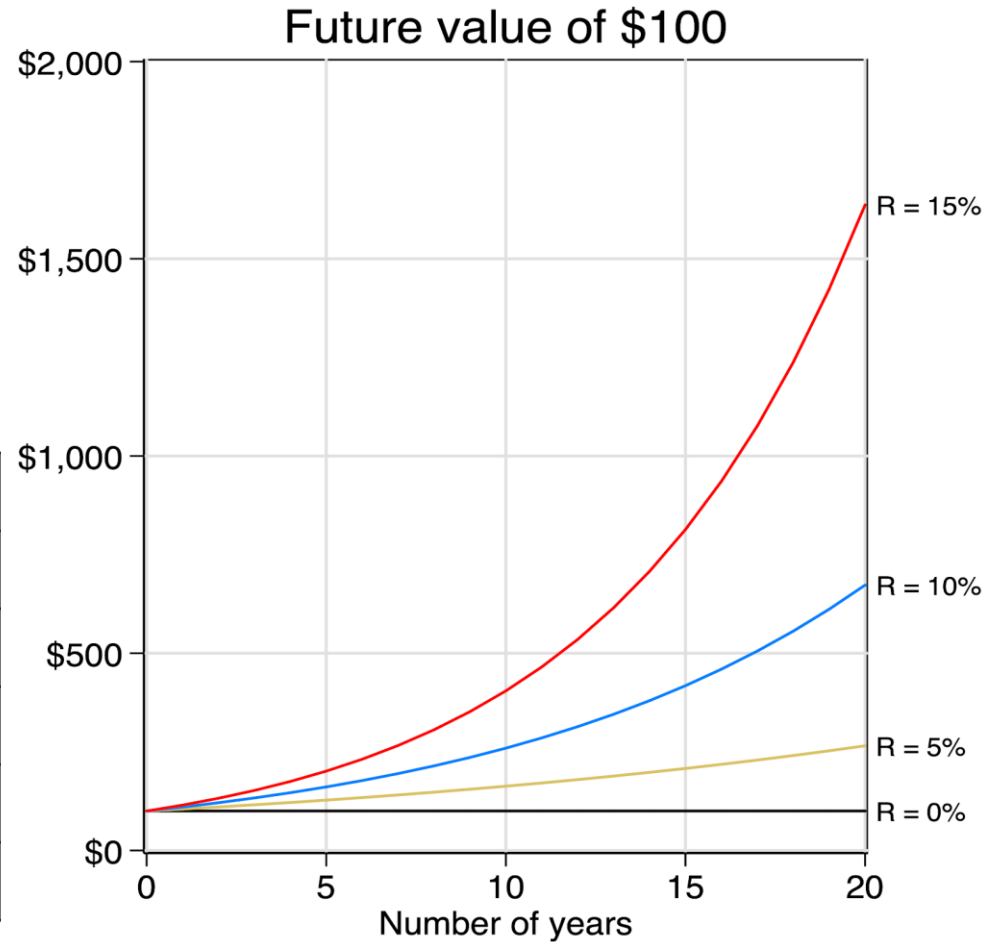
- $FV = PV \times (1 + r)^T$
- Future Value (FV) measures the nominal future sum of money that a given sum of money today is “worth” at a specified time in the future assuming a certain rate of return.
- The Future Value consists of the Present Value (the principal), simple Interest, and interest on interest.

Future Value Formula

The future value of investing an amount PV for T years when the interest rate is R is

$$FV = PV \times (1 + R)^T$$

	0%	5%	10%	15%
0	100	100	100	100
5	100	128	161	201
10	100	163	259	405
15	100	208	418	814
20	100	265	673	1,637



New York Real Estate Bargain

- The Dutch West India Company dispatched the first permanent settlers to Manhattan Island in 1624. They established the town of New Amsterdam as more settlers arrived.
- In 1626, the fledgling town's governor, Peter Minuit, bought Manhattan—meaning “Island of Hills”—from the Canarsie Indians for 24 dollars' worth of beads and trinkets. New Yorkers often call this the last real estate bargain in New York.
- How good was it a bargain if the interest rate was 5%?
- What is the interest rate was 6%?

New York Real Estate Bargain

- Suppose interest rates are 6%, at the beginning of 2026

$$FV = PV \times (1 + r)^T = \$24 \times (1.06)^{400} = \$318,095,369,845$$

- Suppose rates are 7%:

$$FV = PV \times (1 + r)^T = \$24 \times (1.07)^{400} = \$1.36 \times 10^{13}$$

- A future value of 13.6 trillion!

Present Value (PV)

- How much do you need to invest today, with an interest rate of r , to have \$FV next year?

$$PV = \frac{FV}{(1 + r)^t}$$

- Discount factor or the Present Value factor is $\frac{1}{(1+r)^t}$
- Present value factor tells us how to convert future value to present value
- We say we calculate PV by *discounting* the FV at a *discount rate* r

Present Value Example

- To receive \$1000 in one year, how much should I invest today at a rate of 5%?

$$PV = \frac{FV}{(1 + r)^t} = \frac{1000}{(1 + 5\%)^1} = 952.38$$

- To receive \$1000 in two years, how much should I invest today at a rate of 5%?

$$PV = \frac{FV}{(1 + r)^t} = \frac{1000}{(1 + 5\%)^2} = 907.03$$



Yield

- Now let's introduce another term, the yield.
- At what hypothetical constant rate does \$P V grow to exactly \$F V in a span of T years?

$$r = (FV/PV)^{1/T} - 1$$

- For a bond, the yield also called yield-to-maturity.
- For a single payment bond, the interest rate and the yield are the same thing. More complicated for multiple-payment bonds.

Yield

- Microsoft seeks to raise \$1.3 billion from investors in exchange for promising to repay \$2.5 billion on 25 years.
- What yield is Microsoft offering?

Yield

- Microsoft seeks to raise \$1.3 billion from investors in exchange for promising to repay \$2.5 billion on 25 years.
- What yield is Microsoft offering?
- $r = (2.5 / 1.3)^{(1/25)} - 1 = 2.65\%$
- A single payment: the interest rate and the yield are the same thing

Time

- Your aunt is 40 years old and has \$500,000 saved for retirement. She wants to retire with \$4 million without any additional savings.
- If the rate of return on her savings is 8%, at what age can she retire?

Time

- Your aunt is 40 years old and has \$500,000 saved for retirement. She wants to retire with \$4 million without any additional savings.
- If the rate of return on her savings is 8%, at what age can she retire?
- $t = \log(4,000,000 / 500,000) / \log(1.08) = 27$ years
- She can retire at 67
- Doubling three times ($0.5 \rightarrow 1 \rightarrow 2 \rightarrow 4$) at 8%: about 9 years each



Recap

- In general

$FV = PV \times (1 + r)^t$	$r = \left(\frac{FV}{PV}\right)^{\frac{1}{t}} - 1$
$PV = \frac{FV}{(1 + r)^t}$	$t = \frac{\log\left(\frac{FV}{PV}\right)}{\log(1 + r)}$

- Given any 3, you can find the 4th variable

Practice Questions

- If you invested \$1,000 and received \$1,108 after 3 years, what was the interest rate?
- If you invested \$1,000 at 6% and received \$1,791, for how long did you invest?
- You want to have \$1,000 in four year's time. Interest rates are 2%. How much do you need to invest today?

Practice Questions

- If you invested \$1,000 and received \$1,108 after 3 years, what was the interest rate?
- If you invested \$1,000 at 6% and received \$1,791, for how long did you invest?
- You want to have \$1,000 in four year's time. Interest rates are 2%. How much do you need to invest today?
- $r = (1,108/1,000)^{(1/3)} - 1 = 3.48\%$
- $t = \log(1.791) / \log(1.06) = 10 \text{ years}$
- $PV = 1,000 / 1.02^4 = \$923.85$



Multiple Cash Flows

- The present value of a set of cash flows CF_t received in $t = 1, 2, \dots, T$ years is equal to the sum of the present values of the individual cash flows:

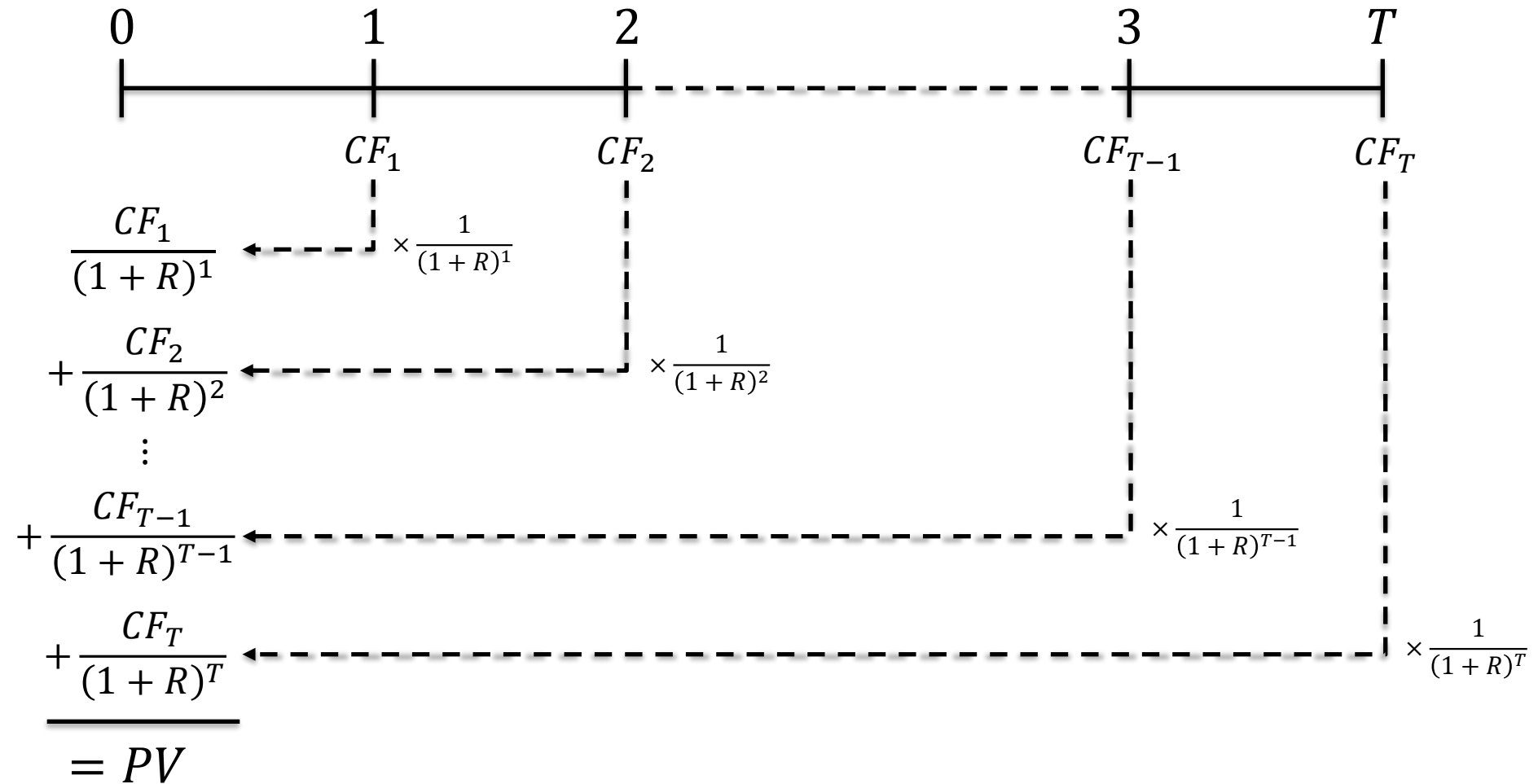
$$PV = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \dots + \frac{CF_T}{(1+r)^T} = \sum_{t=1}^T \frac{CF_t}{(1+r)^t}$$

- Similarly, the future value of a set of cash flows CF_t is:

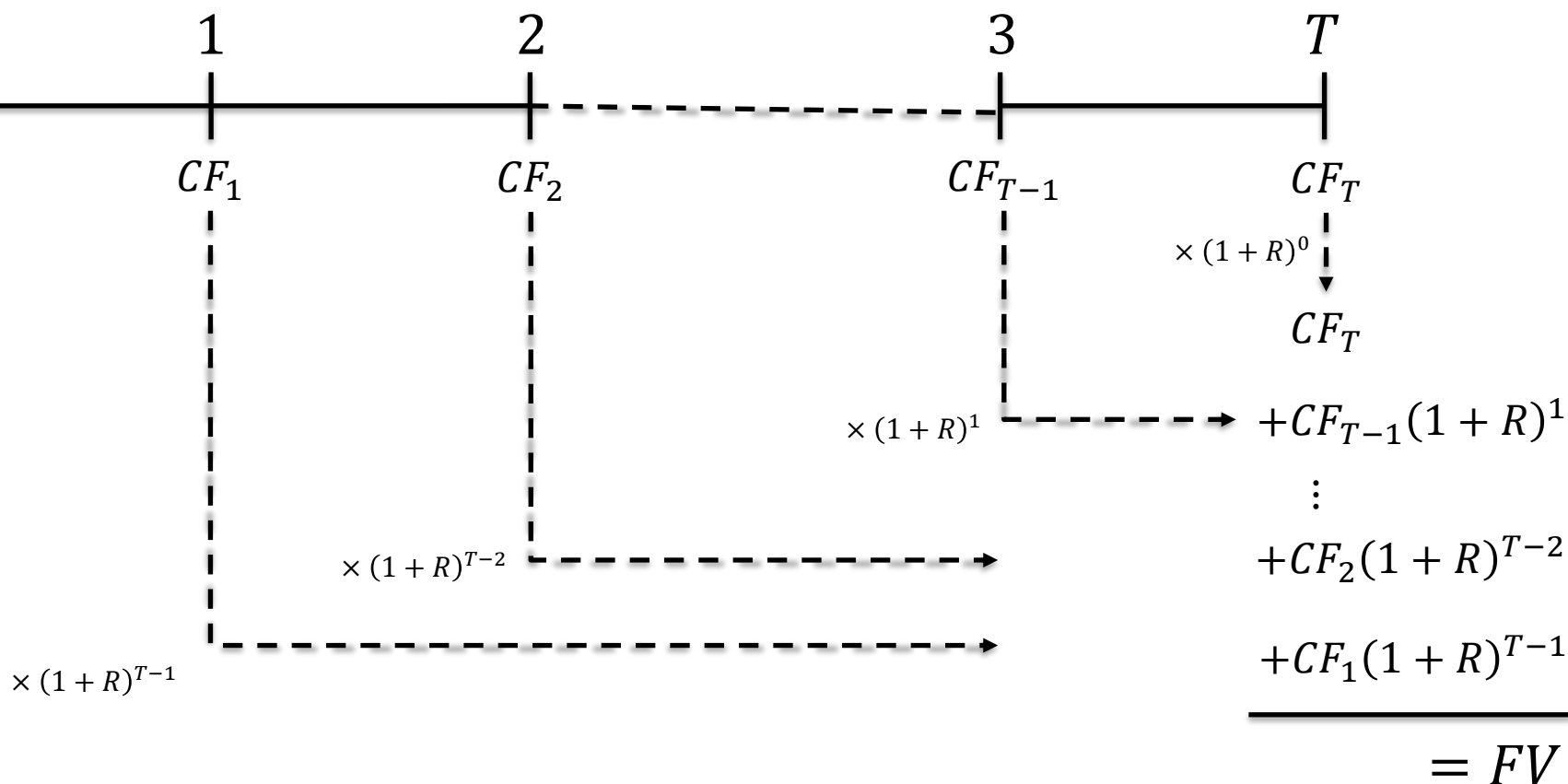
$$FV = CF_1(1+r)^{T-1} + CF_2(1+r)^{T-2} + \dots + CF_T = \sum_{t=1}^T CF_t(1+r)^{T-t}$$

- Note that, as before, $FV = PV(1+r)^T$

Multiple Cash Flows - PV



Multiple Cash Flows - FV



PV and FV of Multiple Cash Flows

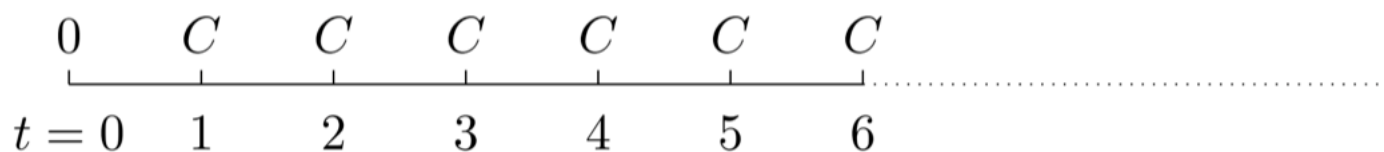
- Your company is considering buying an office building that it plans to rent out for \$40 thousand per year (net of expenses) for 5 years, then sell for \$500 thousand
- How much should your company willing to pay for the building if the appropriate discount rate is 6%?
- How much cash will your company have from collecting rent and selling the building after 5 years? Assume your company earns 6% interest on its cash

PV and FV of Multiple Cash Flows

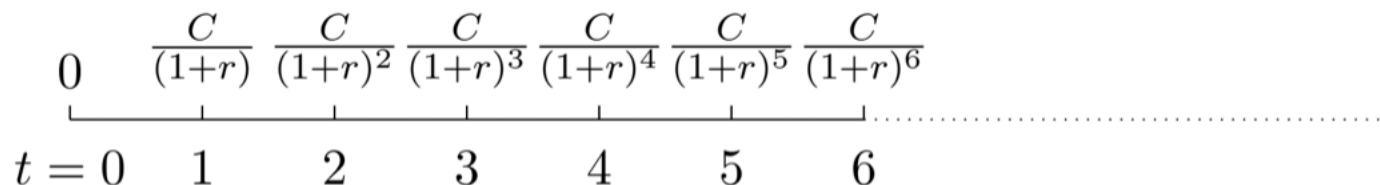
- Buy an office building: rent \$40k a year for 5 years, then sell for \$500k. Discount rate 6%.
- $PV = \sum 40/1.06^t + 500/1.06^5 = \$542.1k$
- FV after 5 years = $\sum 40 \cdot 1.06^{(5-t)} + 500 = \$725.5k$
- Check: $542.1 \times 1.06^5 = 725.5$ — one value, two dates

Perpetuities

- Let's look at a couple of special fixed-income assets.
- A perpetuity or a consol is a bond that pays a fixed payment C forever. No principal payment. By convention, the payment in the current period has already happened.
- PV: Draw the timeline of payments. Note that our definition assumes the payment for $t=0$ has already happened.



- Calculate the present value of each of these payments:



Perpetuity Timeline

- Therefore, the present value of the entire stream of payments is:

$$PV = \sum_{i=1}^{\infty} \frac{C}{(1+r)^i} = C \times \sum_{i=1}^{\infty} \frac{1}{(1+r)^i} = \frac{C}{1+r} \sum_{i=0}^{\infty} \left(\frac{1}{1+r} \right)^i$$

- By formula for geometric sums

$$PV = \frac{C}{1+r} \times \frac{1}{1 - \frac{1}{1+r}} = \frac{C}{r}$$



Perpetuity Example

That Debt From 1720? Britain's Payment Is Coming

After that financial crash in 1720, called the [South Sea Bubble](#), the British government was forced to undertake a bailout that eventually left several million pounds of debt on its books. Almost three centuries later, Britons are still paying interest on a small part of that obligation.

George Osborne, the chancellor of the Exchequer, said this month that in 2015 [Britain](#) would repay part of the country's debt from World War I, and that he wanted to pay off other bonds for debt incurred in the 18th and 19th centuries.

The debt originating in part from the South Sea Bubble, the oldest still on the books, was consolidated into bonds issued in 1853, and those who now own them receive an annual payout of 2.5 percent.



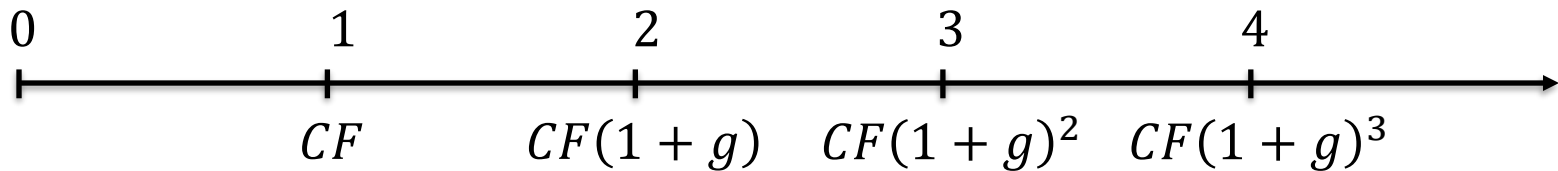
Perpetuity: Example

- How much is a preferred stock worth if it pays \$10 dividend and if the discount rate is 8%?
- If I invest \$125 at 8%, I will earn \$10 ($= 0.08 \times 125$) interest in a year. If I then re-invest the \$125, I will earn another \$10 in two years, and so on. The cash flows are the same as the preferred stock, so the preferred stock is worth \$125
- The value of a perpetuity satisfies $R \times PV = CF$ or

$$PV = \frac{CF}{R}$$

Constant Growth Perpetuities

- What if the cash flows of the perpetuity grow at a constant rate g ?



- Ex: common stock pays \$10 dividend starting in year 1 that grows 4% a year. The discount rate is still 8%.
- Now I need to invest \$250 so that the interest of \$20 ($=0.08 \times 250$) covers both the \$10 dividend and another \$10 to grow the principal by 4%
- So, a constant-growth perpetuity satisfies $R \times PV = CF + g \times PV$
or $PV = \frac{CF}{R - g}$

Perpetuity Example

- Consider a perpetuity that pays \$1,000 forever, however, only starting 10 years from now. Interest rates are 5%. What is the PV?

Perpetuity Example

- Consider a perpetuity that pays \$1,000 forever, however, only starting 10 years from now. Interest rates are 5%. What is the PV?
- A perpetuity formula values it one period before the first payment
- At $t = 9$: $PV_9 = 1,000 / 0.05 = \$20,000$
- Today: $PV_0 = 20,000 / 1.05^9 = \$12,892$

Annuities

- Definition: Pays a fixed cash flow C for T periods (starting at time 1)

$$PV = \sum_{i=1}^T \frac{C}{(1+r)^i} = C \sum_{i=1}^T \frac{1}{(1+r)^i} = \frac{C}{(1+r)} \sum_{i=0}^{T-1} \left(\frac{1}{1+r} \right)^i$$

- By formula for geometric sums:

$$PV = \frac{C}{1+r} \frac{1 - \left(\frac{1}{1+r} \right)^T}{1 - \frac{1}{1+r}} = \frac{C}{r} - \frac{C/(1+r)^T}{r} = C \left[\frac{1 - \frac{1}{(1+r)^T}}{r} \right]$$

$$PV = C \left[\frac{1 - \frac{1}{(1+r)^T}}{r} \right]$$

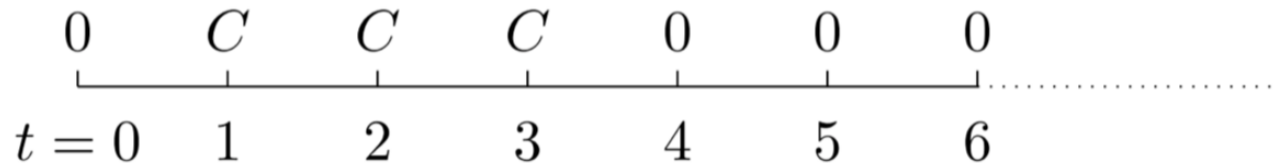
↑
Payment

↑ PV factor

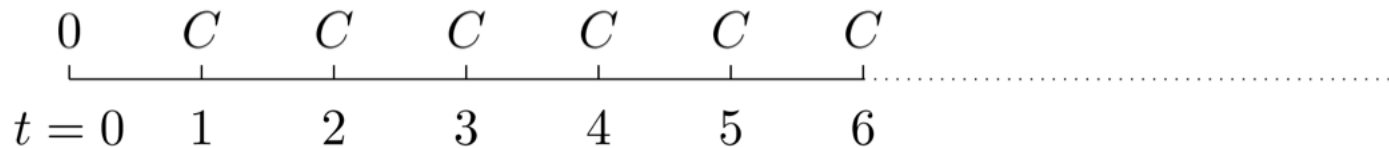


Annuities

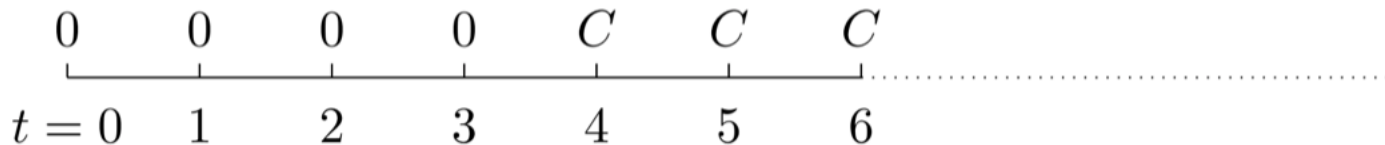
- Definition: Pays a fixed cash flow C for T periods (starting at time i)
- Consider the following annuity



- Also consider perpetuity A

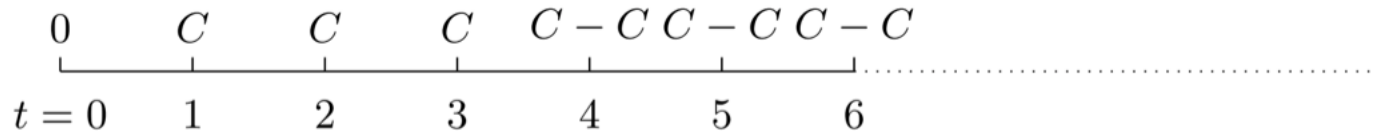


- And perpetuity B



Annuities

- Now consider a portfolio that earns the cash flows from perpetuity A, but has to pay the cashflows of perpetuity B.
- Let's draw the timeline of payments



- These are exactly the payments of the annuity!
- Two portfolios with the same cashflows must have the same price. The PV of the perpetuity must be the PV of perpetuity A, minus the PV of perpetuity B.
- The PV of this annuity is therefore

$$PV = \frac{C}{r} - \frac{C}{r} \frac{1}{(1+r)^3} = C \left[\frac{1 - \frac{1}{(1+r)^3}}{r} \right]$$



Annuity: Example

- Which car can you afford? If
 - You have no large amount of cash
 - You can afford \$632 per month
 - You can borrow at an interest rate of 1% per month
 - You want to have paid the loan in full in 48 months



$$PV = c \left[\frac{1}{r} - \frac{1}{r(1+r)^t} \right]$$

$$= 632 \left[\frac{1}{0.01} - \frac{1}{0.01(1+0.01)^{48}} \right] = 24,000$$



A Closer Look at the Frequency of Computing

- Earlier, we saw the power of compounding over long periods of time, but the effects of compounding also increase with the frequency
- Lenders are required by law to report the annual percentage rate (APR)
- $APR = (\text{rate per period})(\# \text{ of periods per year})$
- For example, a loan that charges 1% per month:
 $APR = 1\% * (12) = 12\%$
- Is 12% per year the “real” cost?
- How do you make a loan seem cheaper?

APR and EAR

- Effective annual rate (EAR): If interest is compounded m times a year, EAR is:

$$EAR = \left(1 + APR/m\right)^m - 1$$

- Which loan is the cheapest?
 - 10% compounded semi-annually
 - 10% compounded quarterly
 - 10% compounded daily

Continuous Compounding

- Consider increasingly frequent compounding: annually, quarterly, daily, every second,...
- When compounding happens “all the time,” it is called continuous compounding
- What happens to the EAR?

$$EAR = e^{APR} - 1$$

- What happens to the FV?

$$FV = \lim_{N \rightarrow \infty} PV \times \left(1 + r/N\right)^{Nt} = PV \times e^{rt}$$

- Similarly, $PV = FV e^{-rt}$



Continuous vs Annual Compounding

- Suppose $r=5\%$ and $PV=\$1,000$
- Annual compounding
 - 1 year?
 - 2 years?
 - T years?
- Continuous compounding
 - 1 year?
 - 2 years?
 - T years?



Example: Payday Loans

- Quick Cash offers 14-day loans to cash-strapped borrowers at a price of \$117 per \$100 lent. The loan is automatically rolled over at the same terms until repaid
- What APR is Quick Cash required to report?
- What is the effective annual rate of the advance?

Example: Payday Loans

- Quick Cash offers 14-day loans at \$117 per \$100 lent, rolled over until repaid
- What APR must Quick Cash report? What is the effective annual rate?
- Rate per 14 days = 17%. $APR = 17\% \times 365/14 = 443\%$
- $EAR = 1.17^{(365/14)} - 1 \approx 5,894\%$
- APR ignores compounding — here that is the whole cost

Summary

- A cash flow means nothing until you say when it arrives
- FV, PV, r and t : given any three, solve for the fourth
- Perpetuities and annuities are shortcuts, not new ideas
- APR is a quote; EAR is what you actually pay
- Next class: Introduction to Bonds (BMAE 3.1) — a bond is a cash flow stream

Thanks

